National Physical Laboratory

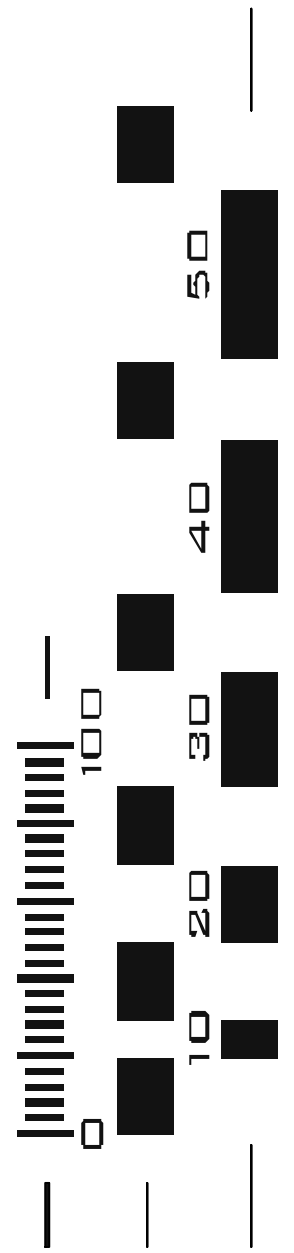
Guide to the Expression of Uncertainty in Measurement (GUM) and its supplemental guides

Maurice Cox
National Physical Laboratory, UK

maurice.cox@npl.co.uk

http://www.npl.co.uk/ssfm/index.html

IAJapan, Tokyo, March 2003



Outline

JCGM

Current GUM

GUM revision

Two phases of uncertainty evaluation

Supplemental guides

Joint Committee for Guides in Metrology (JCGM)

JCGM comprises

Bureau International des Pois et Mesures	(BIPM)
International Electrotechnical Commission	(IEC)
International Federation of Clinical Chemistry	(IFCC)
International Organisation for Standardisation	(ISO)
International Union of Pure and Applied Chemistry	(IUPAC)
International Union of Pure and Applied Physics	(IUPAP)
International Organisation of Legal Metrology	(OIML)
International Laboratory Accreditation Cooperation	(ILAC)

Body responsible for GUM revision

JCGM/WG1, Measurement Uncertainty

NPL

Guide to the Expression of Uncertainty in Measurement

GUM published by ISO in mid-1990s

Widely used and respected

Recognised by many as the master document on uncertainty

Embraces many aspects of uncertainty evaluation

But … some deficiencies and limitations identified

Process of GUM revision under way

NPL

Why revise the GUM?

Improve ease of use

Give added value

Address limitations

Politics

Continue to promote current GUM

- Enormous investment by organisations
- Make no change to GUM *per se*

Added value through supplemental guides

The two phases of uncertainty evaluation

1. **Formulation**
   - Develop (mathematical or other) model
   - Assign input probability distributions

2. **Calculation**
   - Derive output probability distribution
   - Estimate measurand and standard uncertainty
   - Provide coverage interval for measurand
     - E.g., at 95% level of confidence

Current GUM

Commonest implementation of GUM: LPU + CLT

[GUM Clause 8]

Best estimate *y* of measurand *Y*

Model evaluated at best estimates x_i of input quantities X_i

LPU: Law of Propagation of Uncertainty

Combination of standard uncertainties $u(x)$ (and covariances) of input quantities → standard uncertainty $u(y)$ of y

CLT: Central Limit Theorem

Assign a Gaussian (or "*t*") distribution to the measurand

Current GUM

- Implementations
- Higher-order LPU possible
  - Difficult to implement
  - GUM provides formula for mutually independent case
    - Assumes Gaussian
  - No GUM formula for mutually dependent case

Software packages often apply LPU + CLT

Checks for applicability?

NPL

Supplemental guides

1. Propagation of distributions
2. More than one measurand
3. Conformity assessment
4. Modelling
5. Calibration curves

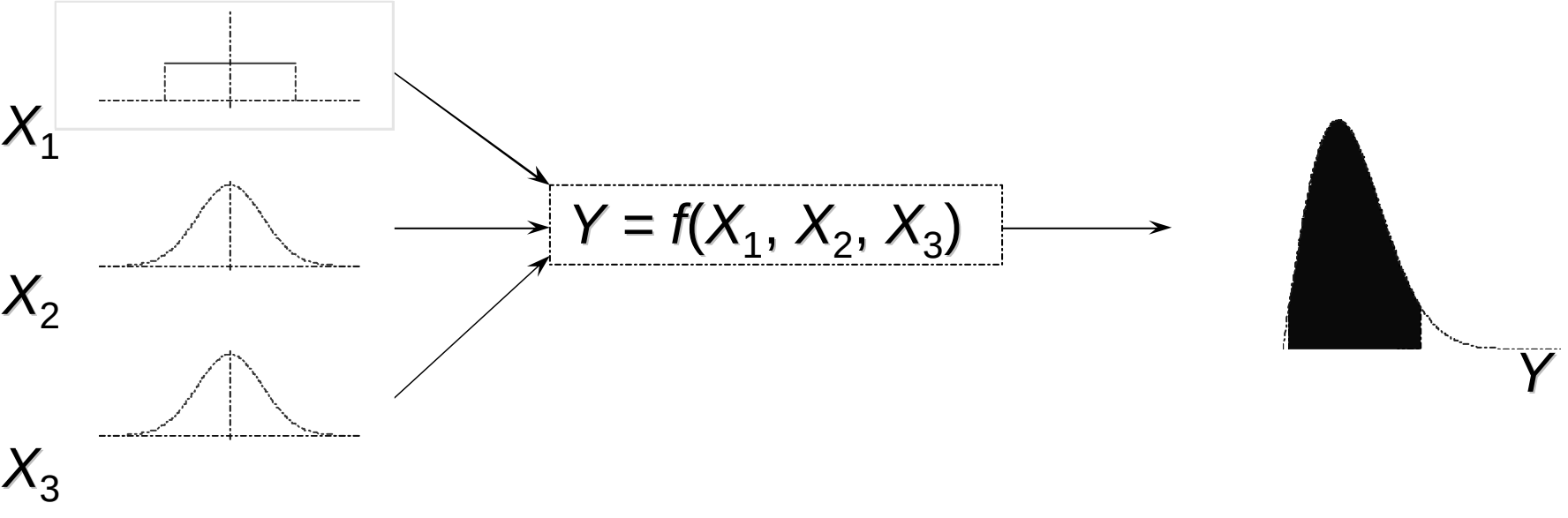
1. Propagation of distributions

Required to ensure valid solution when

- **Contributory uncertainties arbitrarily large**
- **Contributory uncertainties not comparable in size**
- **Very few contributory uncertainties**
- **Models highly nonlinear or complicated**
- **Distributions asymmetric**

NPL

1. Propagation of distributions



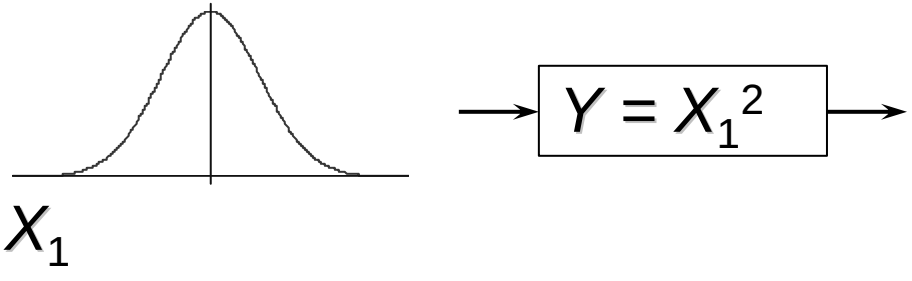
Recommended implementation procedure:

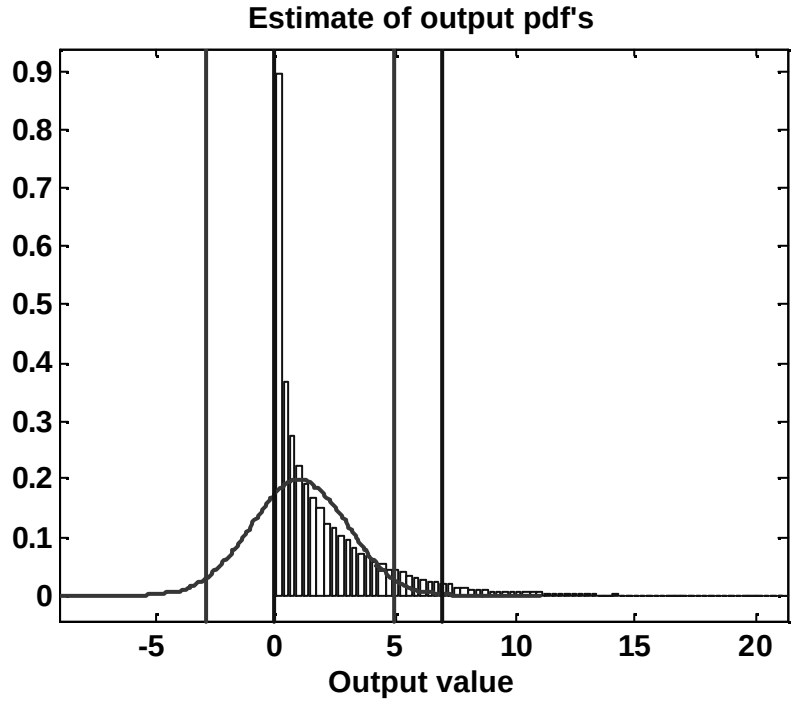
Monte Carlo simulation

- Very simple
- Avoids partial derivatives!

NPL

1. Very simple non-linear model: $Y = X_1^2$





$x_1 = 1, \quad u(x_1) = 1$

Normal input does not imply normal output when model is non-linear

Applicability of Central Limit Theorem!

1. Validation of uncertainty calculations

Use MCS to validate LPU + CLT

Favourable comparison

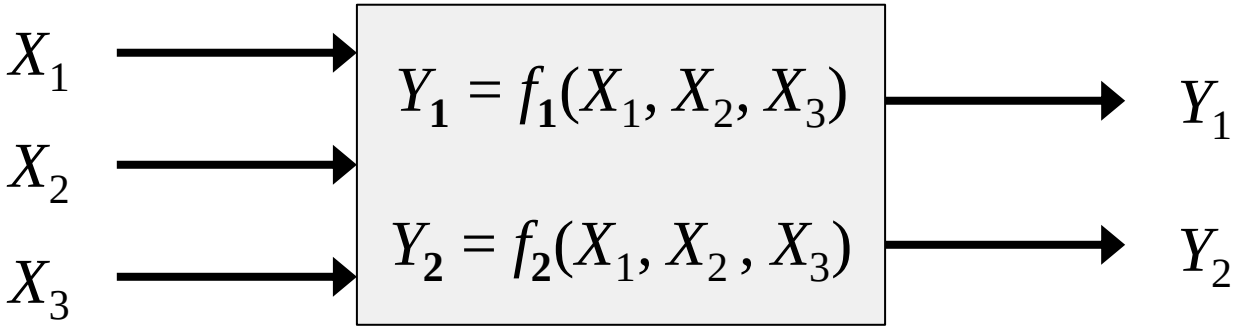
- **LPU + CLT satisfactory for application**

Unfavourable comparison

- **Either detailed investigation required**
- **Or use MCS instead for uncertainty calculation**

NPL

2. More than one measurand



Input quantities X_1, X_2, …

Values, standard uncertainties and covariances

Output quantities Y_1, Y_2, …

Values, standard uncertainties and covariances

Y_1, Y_2, … almost invariably correlated

2. More than one measurand

Generalisations of the GUM formulae

Matrix-vector expressions

- **Compact formulae** — $E.g.,\ V_y = J_x V_x J_x^T$
- **Suitable for software implementation**

Provision of best estimates of Y_1, Y_2, …, and standard uncertainties and covariances

Coverage regions under consideration!

NPL

3. Uncertainty in conformance testing

Industrial inspection: tolerance zones

Regulations: Limit values

Process and inherent variability

Decisions based on measurement

3. Uncertainty in conformance testing

Accounting for measurement uncertainties

Acceptance and rejection of components

Repercussions of surpassing regulatory limits

Decision rules

Consumer’s and producer’s risks

NPL

4. Modelling

The formulation phase of uncertainty evaluation

Physical, empirical and hybrid models

Functional and probabilistic models

Assignment of probability distributions to input quantities

Classification of models

- Explicit or implicit
- Real or complex-valued
- 1 or more than 1 measurand

} $2 \times 2 \times 2 =$ 8 categories

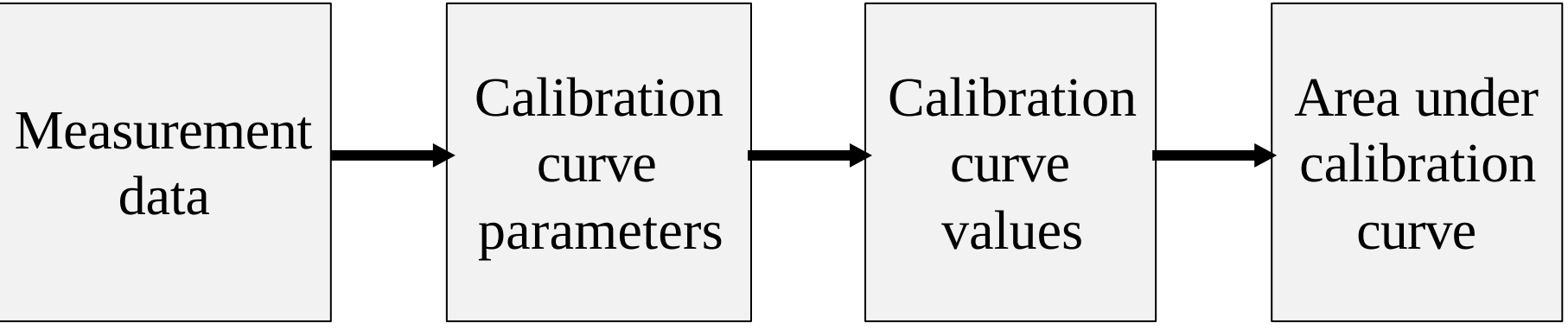
NPL

4. Modelling

Multistage models

Managing multistage models

Example: calibration curves



5. Least squares adjustment

Least-squares modelling

Widespread in metrology

Associated uncertainty evaluation

Not well understood

Calibration curves a very important application

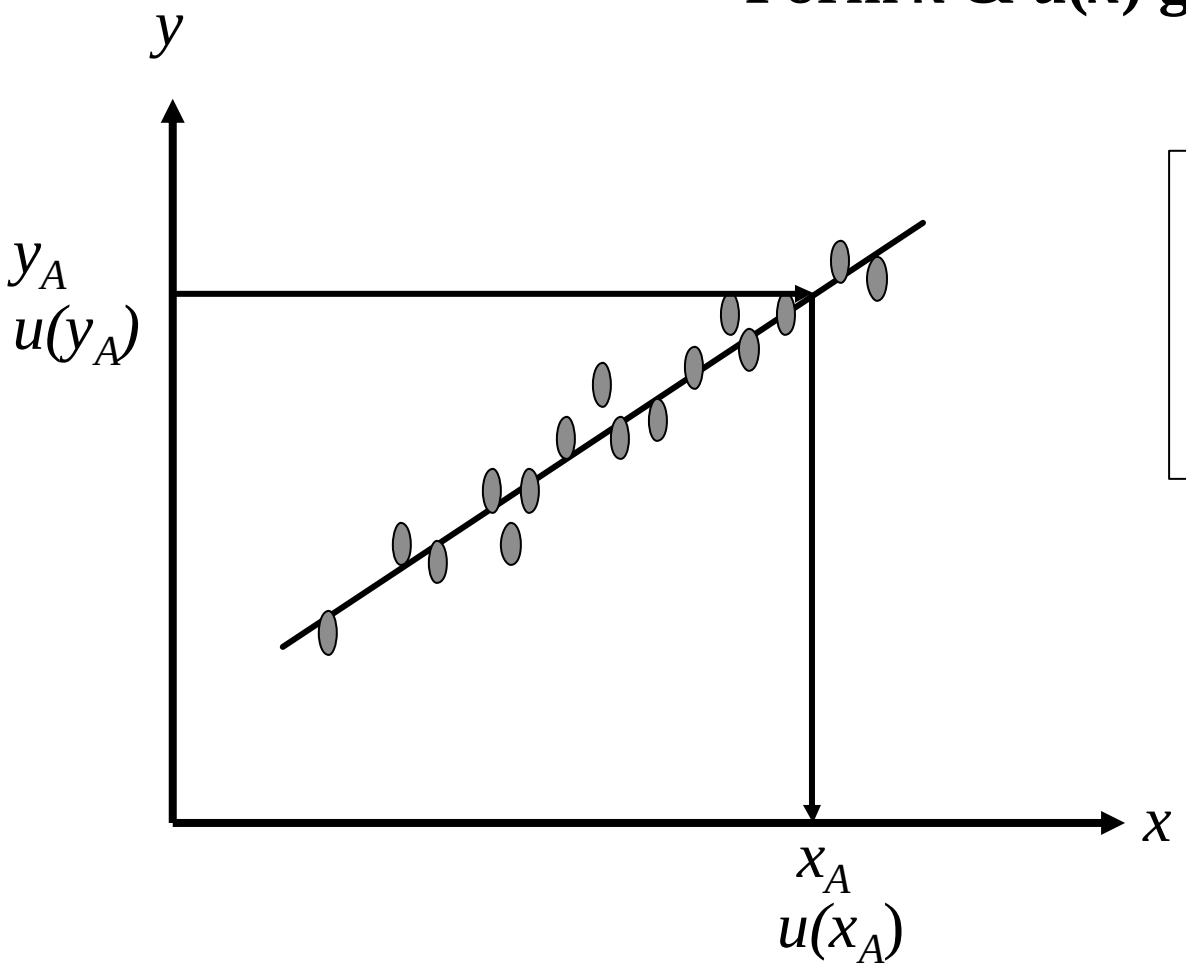
Response vs. stimulus

Generation of curve from standards (response values)

Use of curve to obtain stimulus values

5. Calibration curves

Given stimuli (standards x_i & $u(x_i)$)
Measure responses y_i & obtain $u(y_i)$
Derive regression line (curve) $y = f(x)$
Form x & $u(x)$ given y & $u(y)$



Respect stated uncertainties and known mutual dependencies

JCGM and ISO/TC69 link

Conclusion

Respect investment in common use of GUM

Give added value to current GUM

Two phases of uncertainty evaluation

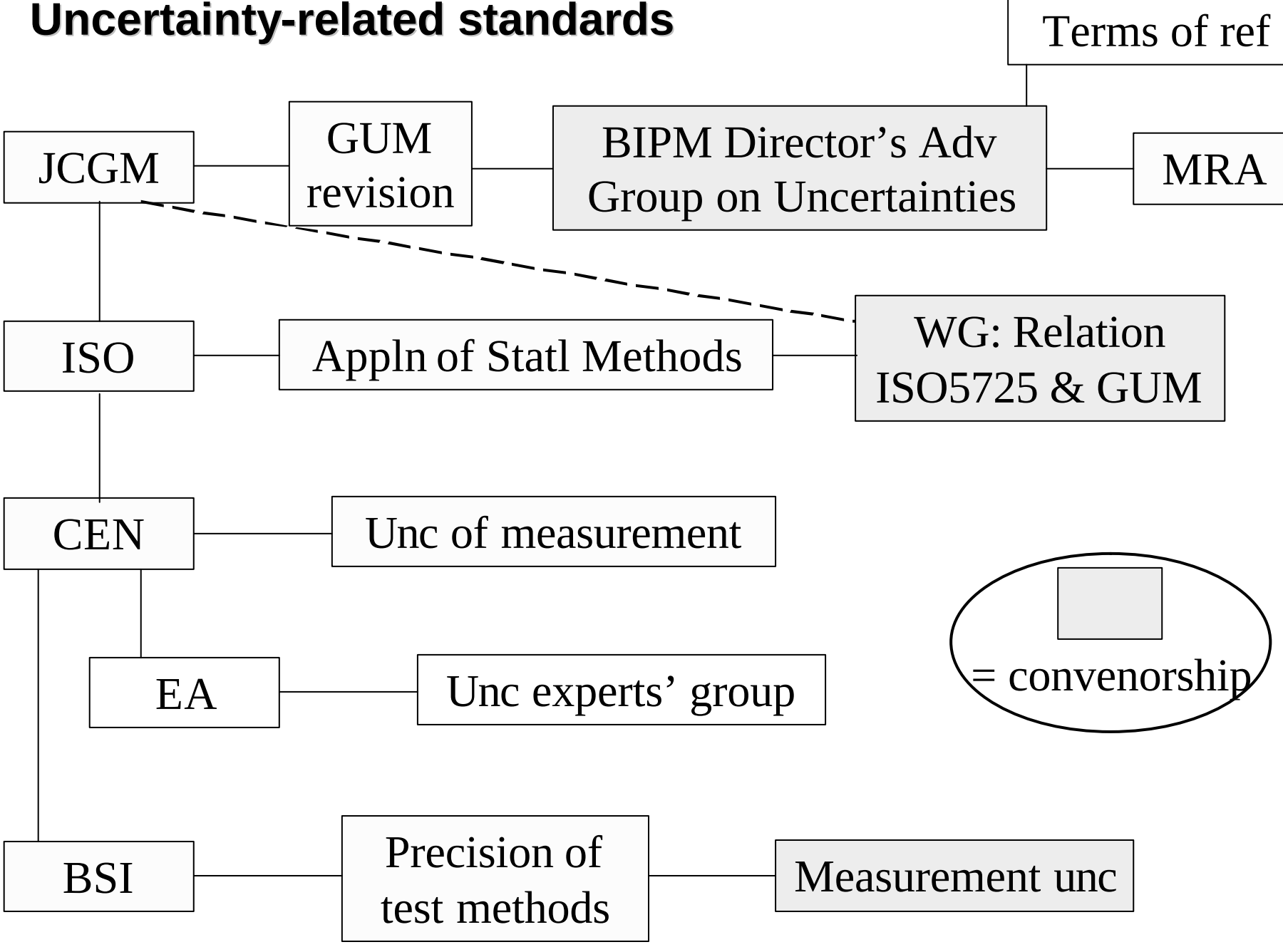
Supplemental guides

1. **Propagation of distributions**
2. **More than one measurand**
3. **Uncertainty in conformance testing**
4. **Modelling**
5. **Least-squares adjustment**

NPL

Optional slides

Uncertainty-related standards



Other new work items in JCGM

Introductory document on uncertainty

Introduce the GUM and its supplemental guides

“Industrial” guide to uncertainty

NPL

Introductory document on uncertainty

Possible scope

- An introduction to measurement uncertainty
- An introduction to the GUM
- An introduction to GUM supplemental guides

Introductory document on uncertainty

Emphasise intention of the GUM

Provide an estimate (the measurement result) of a quantity of interest (the measurand)

Provide a coverage interval about the estimate that can be expected to contain a specified large fraction of the possible values of the measurand

Means of achieving aim

Given

- **The effects on which the measurand depends**
- **Probability distributions assigned to these effects**
- **A rule/functional relationship/model for expressing the measurand in terms of these effects**

Determine probability distribution for the measurand →

- **the measurement result**
- **the associated standard uncertainty**
- **a coverage interval for the measurand**

NPL

Audience

Many bodies, including

- **Academia**
- **Industry**
- **Accreditation bodies**

Approach

Describe scope of GUM and supplemental guides

Help to the reader to navigate through them

Three considerations

- Concepts
- Formulation
- Implementation

Particular GUM implementation

- LPU + CLT

NPL

Discussion