

Generating and Using Control Charts

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1.0 INTRODUCTION

Types of Control Charts

There are four major types of control charts. The first type, the x-chart (and related xbar, r, and s-charts) is a generic and simple control chart. There are advantages to the xbar, r, and s-charts, and details are available in many statistical texts and will not be covered here.. The x-chart offers the benefit of ease of construction and understanding, at some potential loss of detail and rigor. The x-chart is designed to be used primarily with "variables" data, which are usually measurements such as the length of an object, the time it takes to complete a process, or the number of objects produced per period. Examples at DOE facilities may include square meters of contaminated area cleaned per quarter, liters of waste processed, or time to complete a maintenance work order.

In addition to the x-chart there are three specialized types of control charts, the p-chart, c-chart, and u-chart. These charts are used when the data being measured meet certain conditions (or attributes). The p-chart is used with "binomial" data. P-charts are used for results of go-no go tests, such as percent of work orders completed within budgeted cost. In this case, a work order is either completed within budget or not ("go-no go"). P-charts have the advantage of taking into account the sample size (the number of work orders completed) and accounting for the high level of random fluctuations when the sample size is small (very few work orders completed).

The c-chart is used for "Poisson" processes. These are used with random arrival models, or when "counting" attributes. This type of chart, for example can monitor the number of "defects" in each of many equal samples (constant sample size). Occurrence Reporting data (number of reports per month) empirically appear to fit the "Poisson" model, and the c-chart is recommended when charting occurrence report counts.

The u-chart is used when counting "defects" per sample when the sample size varies for each "inspection." A good example at DOE facilities is the number of Total Recordable Cases (TRC) per 200,000 hours. The number of cases is counted for fixed time intervals, such as monthly or yearly, but the sample size (number of man-hours worked during each time interval) changes.

2.0 INITIAL CONSTRUCTION OF A RUN CHART

A run chart is a preliminary display of the performance indicator data. It is simply the raw data for each time interval plotted on a chart. Average and control limits have not yet been added.

2.1 Define and Document the Source of Data

Document what date will be associated with each data point. This is very important when more than one date could be associated with the indicator data. An example of several dates being associated with indicator data is that of an occurrence report which has a discovery date, a categorization date, and a reporting date.

Document what subset of data will be used from the Indicator Data.

EXAMPLE:

Number of New Occurrence Reports Written per Month

Source: Occurrence Reporting and Processing System (ORPS) database

Time Interval: Monthly

Date Used: The Categorization Date will be used to decide in which month the data fall

Sub-Set: All Occurrence Reports with an "SR" prefix

2.2 Select Axes for Graph

The x-axis will be time, generally by Quarter, Month, or Week.

Scaling of the x-axis should be strictly linear. Do not mix time intervals on the same graph (such as plotting past data by fiscal year, but current data by month). The y-axis will be the data values corresponding to each time interval. Scaling of the y-axis should generally be from zero through the highest data value (or highest control limit value).

Shifting the y-axis such that it starts at a value higher than zero is to be avoided as most readers expect graph scales to be zero-based and linear. When the datum value plotted on the graph appears to be twice as large as the previous value, most people assume that the data has in fact doubled. Starting the scale at a non-zero value skews this perspective, and should be avoided unless absolutely necessary to see trends in the data.

2.3 Special Criteria

Indicator Data should be "independent" from one time interval to the next. Each time interval's datum value should not be influenced by the previous time interval's value. For example, this month's inventory in a warehouse is highly dependent upon the ending inventory from the previous month. Generally, rate data (Items per Time, i.e., number of new receipts arriving at the warehouse each month) is independent.

2.4 Gather and Plot Data

Gather the data for each time interval.

Establish the method that will be used for creating the charts, e.g., graph paper, or computer graphing routine such as Excel ® Spreadsheet.

Set up and label the axes.

Plot each period's datum point on the graph.

Connect the points with line segments from point to point.

These steps complete the construction of the Run Chart.

3.0 CONVERT THE RUN CHART TO A CONTROL CHART

Convert the run chart to a control chart by adding three additional lines to the graph: the average (mean) line and the three standard deviation upper and lower control limits. Calculation of the average is similar for all control charts. However, calculation of the control limits depends on the type of control chart.

3.1 Choose the Time Period for Which the Baseline Average and Control Limits will be Calculated

It is best to have at least 25 data points in the baseline time period. If there are less than 25 data points available, then use what data are available. Sometimes it is advantageous to leave a few recent data points outside the baseline so that it is possible to assess the effectiveness of the baseline. Leaving some recent points out of the baseline allows them to be compared to the baseline average and control limits, without having had an influence in the average and control limit calculations. This provides a test of how good the baseline is at predicting future values of the indicator. If these points that were left out of the baseline do not generate any significant trends (per criteria of section 3.7), then this verifies that the baseline is effective.

3.2 Calculate the Arithmetic Average of the Data

The arithmetic average is also known as the mean. The average is the sum of the data divided by the number of data points. This is used as the “baseline average” on the control chart.

NOTE: If the data are a ratio (such as days away from work cases per 200,000 hours), sum all of the numerators (total all the days away from work cases), then sum all of the denominators (total of all the hours), then divide (total number of cases divided by total hours) then multiply by 200,000.

3.3 Draw the Average as a Horizontal Line on the Graph

It is a good idea to label the average with its value and the time period for which it was calculated.

3.4 Calculate the Standard Deviation

The data used to calculate the standard deviation (usually represented by the Greek letter sigma “ σ ”) are the same data used to calculate the average in step two above.

X-chart: There are two acceptable methods to calculate the standard deviation for a x-chart. The first is supported by many computer software programs and statistical calculators. After entering the data into a calculator or computer, determine the "sample standard deviation." This is usually annotated by the Greek letter sigma (s), and subscript ($n-1$), shown as s_{n-1} . Do not use the population standard deviation (usually annotated as sigma n and written as s_n). The second method is to multiply the average range value for pairs of data points by 0.887. The former DOE-STD-1048-92 used this method for calculation of control limits.

X Bar-r-chart: The x bar-r-chart is a more complicated (but sometimes more useful) chart than the x-chart. The advantage of these two charts is that both the central tendency, and the spread of the data are evaluated for trends. The disadvantage is that the data are plotted in subgroups which may be difficult to explain (each data point may represent 5 sequential work items, rather than one month's worth of work items). Also, the concept of plotting the average and the range are more difficult to grasp. The data are divided into sub-groupings of (usually) four or five points. The average of each sub-group is plotted on the x bar chart, and the range (the highest value minus the lowest value) is plotted on the r-chart. The standard deviation for each graph is calculated by determining the average of the range values (signified as $rbar$) per the tables available in statistical handbooks. For most DOE performance measures, the x-chart described above will be sufficient.

C-chart: The standard deviation is the square root of the average. $\sigma = \sqrt{xbar}$

Where: $xbar$ = baseline average

P-chart: The standard deviation is calculated for each datum value. The formula is:

$$\sigma = \sqrt{\frac{pbar(1 - pbar)}{n}}$$

Where: $pbar$ = baseline average and n = number of trials

Note that the standard deviation varies with the number of trials, so the standard deviation must be calculated separately for each datum value. Example - the current baseline average is an 80% success rate. This month had 20 trials with 18 successes (for a rate of 90% for the month). The standard deviation for this month is:

$$\sigma = \sqrt{\frac{0.8(1-0.8)}{20}} = 0.089$$

U-chart: The standard deviation is calculated for each datum value. The formula is:

$$\sigma = \sqrt{\frac{\bar{u}}{n}}$$

Where: \bar{u} = average

and n = number of units

Example - the current TRC baseline average rate is 1.2 cases per 200,000 hours. This month there were two TRC cases and the sample size for the current month was 361,784 hours worked. The TRC rate for the month is 1.11 cases per 200,000 man hours. The standardized sample size for occupational illness and injury tracking is 200,000 hours. The standard deviation for this month is:

$$\sigma = \sqrt{\frac{1.2}{361,784/200,000}} = 0.81$$

3.5 Calculate and Plot the Upper Control Limit

Add three times the standard deviation to the average. This is the Upper Control Limit (UCL). Plot the UCL on the graph.

The UCL will be a horizontal line on x-charts and c-charts.

The UCL will be variable on p-charts and u-charts. Do not plot the UCL on a p-chart if it exceeds 100%.

3.6 Calculate and Plot the Lower Control Limit

Subtract three times the standard deviation from the average. This is the Lower Control Limit (LCL). Plot the LCL on the graph.

The LCL will be a horizontal line on x-charts and c-charts.

The LCL will be variable on p-charts and u-charts. Do not plot the LCL on a p-chart if it is below 0%.

SPECIAL NOTE: If the LCL is negative (less than zero), and the data could not possibly be less than zero, e.g., a negative number of reports written, or a negative time period, then the LCL is not plotted.

3.7 Analyze the Baseline Data

Look for trends in the existing data. The list of criteria below is the formal definition of a "trend."

- Individual points above the Upper Control Limit.
- Individual points below the Lower Control Limit.
- Seven points in a row all above average or all below average.
- Seven points in a row increasing.
- Seven points in a row decreasing.
- Ten out of eleven points in a row all above average or all below average.
- Two out of three points in a row outside of two standard deviations above the average, or two out of three points in a row outside of two standard deviations below the average.
- Four out of five points in a row outside of one standard deviation above the average, or four out of five points in a row outside of one standard deviation below the average.

4.0 TREATMENT OF STATISTICALLY SIGNIFICANT CHANGES

If any of the criteria of 3.7 (above) exist in constructing a new control chart the average control limits will need to be calculated. If any criteria of 3.7 above are developed with new data on a control chart, then the data points identified by the criteria will be circled. As further data are added to the chart consideration should be made for shifting the average and control limits per guidance below.

4.1 A Trend Signal with a Small Number of Points

For purposes of sections 4.1 and 4.3, the difference between a "small number of points" and a "large number of points" is that the "small number of points" does not yet meet the "MW" criteria of section 4.4. Note that it requires at least five data points to meet the "MW" criteria.

If this occurs during the initial construction of a control chart shows a small number of points forming a trend, consider recalculating both the average and standard deviation (which affects the control limits) without those point(s) included. Ideally, the cause for the outlier(s) should be determined and verified that each was a special case that will probably not recur.

After recalculating the average and control limits, look at the remaining data. If the new average line appears to split the remaining data better than the original average (equal data points on either side of the average), then this confirms the credibility of removing the outliers).

After recalculating the average and control limits, further points may have become outliers. In severe cases, this can lead to an endless series of throwing out "outliers" until

very little data are left. In such cases, it is best to revert back to the original average and control limits and circling the points that are part of the trend

4.2 Individual Point(s) Close to the Control Limits

In some cases, individual point(s) may be so close to the control limits that if they were removed from the average and control limit calculations, these point(s) would become outside the control limits. This is a "gray" area. Confirm that this action should be taken prior to taking it.

Confirmation includes either:

- Knowing (from knowledge of the process) that it is a special case and knowing its cause, or
- Review of the average line shows that it is skewed toward the potential outlier when compared with the remaining data.

4.3 A Trend Signal with a Large Number of Points

This condition indicates that there is not a single, stable baseline. The data may need to be split into two or more baseline time intervals. The average and standard deviation (and control limits) need to be calculated for each time interval. Ideally, the cause for the change between each region should be determined. Each time interval must be non-overlapping. That is, do not calculate one time interval as January 98 through July 98 and the next time interval as June 98 through November 98. Try to line up the breaks between time intervals with jumps in the data. It is especially good if the first data point of the region is outside the control limits of the previous region.

4.4 When to Form a New Baseline (“MW Criteria”)

First, do not split data into two baseline time intervals or shift to a new baseline average and control limits unless there has been a trend detected by the trend rules in 3.7.

Experience by the author has found that there are sufficient data for a new baseline following a trend signal when the points form a “M” or a “W” shape – that is, have three changes of direction in the data. This is referred to in this paper as the “MW Criteria.” These changes of direction may be from increasing to decreasing to increasing to decreasing to increasing, or they may be from decreasing to increasing to decreasing to increasing to decreasing.

The series of numbers 1, 5, 6, 3, 2, 7, 4 form a letter “M” shape and would be sufficient for a baseline average and control limits.

The author has not found any substantiated rule for the sufficiency of a baseline in any statistical references. Usually the decision of when there are sufficient data to form a baseline is left to the analyst. However, the aforementioned “MW Criteria” has served

the author well in 17 years of experience. Note that at least five data points are required to form the “M” shape or the “W” shape.

4.5 Seven Points in a Row all Increasing/Decreasing

This condition indicates that a continuous change (ramp) is occurring in the data. There are two schools of thought for these periods:

- Do not add any average or control limits, as the process is not stable. Statistical Process Control is meaningless in this condition.
- Add average and control limits. The Statistical Process Control (SPC) technique verifies that the data are undergoing continuous, significant change. Statistical Process Control can be thought of as a formal "operational definition" for the existence of significant change(s). In this case, the "definition" shows that significant change is occurring. If average and control limits are added, show the continuous change as a series of step changes using average and control limits over multiple, non-overlapping regions.

Considerations are similar to the methods for seven points in a row above/below average. If the regions become too short, simply leave one longer time interval region with points going from below the LCL to above the UCL (or vice-versa).

4.6 Effects on the Standard Deviation

Following removal of outliers or breaking the data into two or more regions, and recalculating a new standard deviation, there should be a decrease in the standard deviation on x-charts. This will cause the control limits to be closer to the average. If there is not an obvious reduction in the standard deviation, splitting the data or throwing out the outlier may not be justified.

5.0 MAINTAINING A CONTROL CHART

5.1 Updating with a New Datum

Extend the average line horizontally for x- and c-charts. Also extend the control limit lines horizontally. For p- and u-charts calculate the control limits based upon current baseline average.

Early data may be dropped if the graph becomes too compressed. However, do not reduce the number of plotted points to less than 25. A philosophy such as “keep the previous two years plus the current year of data on the chart” will keep at least 25 points on the chart (if the chart is plotted with monthly increments).

Document the time interval for which the average and control limits were calculated. Do not recalculate the average each time interval for any of the control charts, and for x- and

c- charts do not recalculate the control limits each time interval.

5.2 Examine the Graph at Each Update for Statistically Significant Trends

See criteria in section 3.7

5.3 If a Statistically Significant Trend is Detected

Following the detection of a trend per section 3.7 on a periodic update of an existing indicator:

- Annotate the trend on the chart with a circle or other label
- Determine the cause
- Determine actions (Corrective or Reinforcing, depending on direction of change) to be taken. "No action required" is also acceptable, if this is a conscious decision
- Display at least one update of the chart with the annotated trend before considering shifting to a new baseline
- Continue to monitor the data for a return to the previous stable baseline, or stabilization on a new baseline.

If future data return to within the control limits and bounce around both sides of the average, leave existing average and control limits in place. The trend was a special case, and appears to either have corrected itself or been corrected for.

If future data remains the same side of the average line, consider calculating a new baseline average and control limits. Overall, the considerations are the same as evaluating an initial baseline. The "MW Criteria" discussed in section 4.4 may be of assistance.

If the decision is made to calculate new average and control limits, the new baseline will likely contain less than 25 data points. Continue to monitor incoming data. If a trend (per section 3.7) develops against the new baseline, consider adding the new points into the baseline calculation and determine if this eliminates the trend. If the trend is eliminated (3.7 no longer met), then shift to the new baseline. If the trend is not eliminated, do not change the baseline and instead treat the new data as a trend.

Never recalculate the average and control limits by incorporating every new datum in every update or routinely at the end of a calendar year or fiscal year. Only calculate a new baseline average and control limits following detection of a trend per the criteria in section 3.7.

5.0 CONTROL CHARTS RESULTS

Use of Control Chart Results

Control charts serve an important function. The control charts include methodology to detect significant change. The charts include a baseline average line, and control limits. The control limits act as alarm values. If a performance measure datum point falls outside of the control limits (or other significant changes are detected using the other criteria listed in section 3.7), then this is a significant event, which demands investigation and action. To reinforce this idea, it is useful to document the "owner" of each performance measure and what management decisions have been made or will be made based upon this measure.

A trend may be viewed as a change in risk. This interpretation is especially useful when using SPC in conjunction with "leading indicators". When a trend exists on a "leading indicator" it is time to assess the effect of the underlying causes of the change on the culture and management systems of the affected organization.

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