Generating and Using Control Charts

Extract from HNF-4931, Rev. 1	Hanford employees should use the current version of HNF-PRO-4294 (which references HNF-4931) found in the Project Hanford Management System docs online

1.0 INTRODUCTION

Types of Control Charts

There are four major types of control charts. The first type, the x-chart (and related xbar, r, and s-charts) is a generic and simple control chart. (There are advantages to the xbar, r, and s-charts, and details are available in many statistical texts.) This Appendix, however, will concentrate on the x-chart, which is the easiest to construct. The x-chart is designed to be used primarily with "variables" data, which are usually measurements such as the length of an object, the time it takes to complete a process, or the number of objects produced per period. Examples at DOE facilities may include square meters of contaminated area cleaned per quarter, liters of waste processed, or time to complete a maintenance work order.

In addition to the x-chart there are three specialized types of control charts, the p-chart, c-chart, and u-chart. These charts are used when the data being measured meet certain conditions (or attributes).

The p-chart is used with "binomial" data. P-charts are used for results of go-no go tests, such as percent of work orders completed within budgeted cost. In this case, a work order is either completed within budget or not ("go-no go"). P-charts have the advantage of taking into account the sample size (the number of work orders completed) and accounting for the high level of random fluctuations when the sample size is small (very few work orders completed).

The c-chart is used for "Poisson" processes. These are used with random arrival models, or when "counting" attributes. This type of chart, for example can monitor the number of "defects" in each of many equal samples (constant sample size). Occurrence Reporting data (number of reports per month) empirically appear to fit the "Poisson" model, and the c-chart is recommended when charting occurrence report counts.

The u-chart is used when counting "defects" per sample when the sample size varies for each "inspection." A good example at DOE facilities is the number of lost or restricted workday cases per 200,000 man-hours. The number of cases is counted for fixed time intervals, such as monthly or yearly, but the sample size (number of man-hours worked during each time interval) changes.

2.0 INITIAL CONSTRUCTION OF A RUN CHART

A run chart is a preliminary display of the performance indicator data. It is simply the raw data for each time interval plotted on a chart. Average and control limits have not yet been added.

2.1 Define and Document the Source of Data

Document what date will be associated with each data point. This is very important when more than one date could be associated with the indicator data. An example of several dates being associated with indicator data is that of an occurrence report which has a discovery date, a categorization date, and a reporting date.

Document what subset of data will be used from the Indicator Data.

EXAMPLE:

Number of New Occurrence Reports Written per Month

Source: INEL Occurrence Reporting System database

Time Interval: Monthly

Date Used: The Categorization Date will be used to decide which month the data point falls

Sub-Set: All Occurrence Reports with an "RL" prefix

2.2 Select Axes for Graph

The x-axis will be time, generally by Quarter, Month, or Week.

Scaling of the x-axis should be strictly linear. Do not mix time intervals on the same graph (such as plotting past data by fiscal year, but current data by month).

The y-axis will be the data values corresponding to each time interval.

Scaling of the y-axis should generally be from zero through the highest data value (or highest control limit value). Shifting the y-axis such that it starts at a value higher than zero is to be avoided as most people expect graph scales to be zero-based and linear. When the datum value plotted on the graph appears to be twice as large as the previous value, most people assume that the data has in fact doubled. Starting the scale at a non-zero value skews this perspective, and should be avoided unless absolutely necessary to see trends in the data.

2.3 Special Criteria

Indicator Data should be "independent" from one time interval to the next. Each time interval's datum value should not be influenced by the previous time interval's value. For example, this month's inventory in a warehouse is highly dependent upon the ending inventory from the

previous month. Generally, rate data (Items per Time, i.e., number of new receipts arriving at the warehouse each month) is independent.

2.4 Gather and Plot Data

Gather the data for each time interval.

Establish the method that will be used for creating the charts, e.g., graph paper, or computer graphing routine such as Harvard graphics, Excel or QUATTROPRO Spreadsheet, etc.

Set up and label the axes.

Plot each period's datum point on the graph.

Connect the points with line segments from point to point.

These steps complete the construction of the Run Chart.

3.0 CONVERT THE RUN CHART TO A CONTROL CHART

Convert the run chart to a control chart by adding three additional lines to the graph: the average (mean) line and the three standard deviation upper and lower control limits. Calculation of the average is similar for all control charts. However, calculation of the control limits depends on the type of control chart.

3.1 Choose the Time Period for Which the Baseline Average and Control Limits will be Calculated

It is best to have at least 25 data points in the baseline time period. If there are less than 25 data points available, then use what data are available. Sometimes it is advantageous to leave a few recent data points outside the baseline so that it is possible to assess the effectiveness of the baseline. Leaving some recent points out of the baseline allows them to be compared to the baseline average and control limits, without having had an influence in the average and control limit calculations. This provides a test of how good the baseline is at predicting future values of the indicator. If these points that were left out of the baseline do not generate any significant trends (per criteria of section 3.7), then this verifies that the baseline is effective.

3.2 Calculate the Arithmetic Average of the Data

The arithmetic average is also known as the mean. The average is the sum of the data divided by the number of data points.

NOTE: If the data are a ratio (such as lost work day cases per 200,000 man-hours), sum all of the numerators (total all the lost work day cases), then sum all of the denominators (total of all the man-hours), then divide (total number of cases divided by total man-hours) then multiply by 200,000.

3.3 Draw the Average as a Horizontal Line on the Graph

It is a good idea to label the average with its value and the time period for which it was calculated.

3.4 Calculate the Standard Deviation

The data used to calculate the standard deviation are the same data used to calculate the average in step two above.

X-chart: There are two acceptable methods to calculate the standard deviation for a x-chart. The first is supported by many computer software programs and statistical calculators. After entering the data into a calculator or computer, determine the "sample standard deviation." This is usually annotated by the Greek letter sigma (s), and subscript (n-1), shown as s $_{n-1}$. Do not use the population standard deviation (usually annotated as sigma n and written as s $_n$). The second method is to multiply the average range value for pairs of data points by 0.887. The former DOE-STD-1048-92 used this method for calculation of control limits.

X Bar-r-chart: The x bar-r-chart is a more complicated (but sometimes more useful) chart than the x-chart. The advantage of these two charts is that both the central tendency, and the spread of the data are evaluated for trends. The disadvantage is that the data are plotted in subgroups which may be difficult to explain (each data point may represent 5 sequential work items, rather than one month's worth of work items). Also, the concept of plotting the average and the range are more difficult to grasp. The data are divided into sub-groupings of (usually) four or five points. The average of each sub-group is plotted on the x bar chart, and the range (the highest value minus the lowest value) is plotted on the r-chart. The standard deviation for each graph is calculated by determining the average of the range values (signified as rbar) per the tables available in statistical handbooks. For most DOE performance measures, the x-chart described above will be sufficient.

C-chart: The standard deviation is the square root of the average.

$$\sigma = \sqrt{\bar{x}}$$

Where: $\bar{x} = average$

P-chart: The standard deviation is calculated for each datum value. The formula is:

$$\sigma = \sqrt{\frac{\bar{p}(1-\bar{p})}{N}}$$

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$$\text{Where: } \overline{p} = \frac{total \text{ } \# successes}{total \text{ } \# trials} \text{ and } N = \# trials \text{ } in \text{ } this \text{ } time \text{ } period$$

Note that the standard deviation varies with the number of trials, so the standard deviation must be calculated separately for each datum value. Example - the current baseline average is an 80% success rate. This month had 20 trials with 18 successes (for a rate of 90% for the month). The standard deviation for this month is:

$$\sigma = \sqrt{\frac{0.8(1-0.8)}{20}} = 0.089 \text{ } or \text{ } 8.9\%$$

U-chart: The standard deviation is calculated for each datum value. The formula is:

$$\sigma = \sqrt{\frac{\overline{u}}{N/std. \text{ } sample \text{ } size}}$$

$$\text{Where: } \overline{u} = \left(\frac{total \text{ } defects}{total \text{ } sample \text{ } size}\right)\left(\frac{standard}{sample \text{ } size}\right); \quad N = \frac{sample \text{ } size \text{ } for \text{ } current \text{ } time \text{ } period}{(i.e., lot \text{ } sample \text{ } size)};$$

$$std. \text{ } sample \text{ } size = 200,000$$

$$\text{and } (for \text{ } occupational \text{ } illness/injury)$$

Example - the current lost work day case rate is 1.2 cases per 200,000 man-hours. This month there were two lost workday cases and the sample size for the current month was 361,784 man-hours worked. The rate for the month is 1.11 cases per 200,000 man hours. The standardized sample size for occupational illness and injury tracking is 200,000 man hours. The standard deviation for this month is:

$$\sigma = \sqrt{\frac{1.2}{(361,784/200,000)}} = 0.814$$

3.5 Calculate and Plot the Upper Control Limit

Add three times the standard deviation to the average. This is the Upper Control Limit (UCL). Plot the UCL on the graph.

The UCL will be a horizontal line on x-charts and c-charts.

The UCL will be variable on p-charts and u-charts. Do not plot the UCL on a p-chart if it exceeds 100%.

3.6 <u>Calculate and Plot the Lower Control Limit</u>

Subtract three times the standard deviation from the average. This is the Lower Control Limit (LCL). Plot the LCL on the graph.

The LCL will be a horizontal line on x-charts and c-charts.

The LCL will be variable on p-charts and u-charts. Do not plot the LCL on a p-chart if it is below 0%.

SPECIAL NOTE: If the LCL is negative (less than zero), and the data could not possibly be less than zero, e.g., a negative number of reports written, or a negative time period, then the LCL is assumed to equal zero.

3.7 <u>Analyze the Baseline Data</u>

Look for trends in the existing data. The list of criteria below is the formal definition of a "trend."

- Individual points above the Upper Control Limit.

- Individual points below the Lower Control Limit.

- Seven points in a row all above average or all below average.

- Seven points in a row increasing.

- Seven points in a row decreasing.

- Ten out of eleven points in a row all above average or all below average.

- Cycles or other non-random patterns in the data.

- Two out of three points in a row outside of two standard deviations above the average, or two out of three points in a row outside of two standard deviations below the average.

- Four out of five points in a row outside of one standard deviation above the average, or four out of five points in a row outside of one standard deviation below the average.

4.0 TREATMENT OF STATISTICALLY SIGNIFICANT CHANGES

If any of the criteria of 3.7 (above) exist in constructing a new control chart the average control limits will need to be calculated.

If any criteria of 3.7 above are developed with new data on a control chart, then the data points identified by the criteria will be circled.

As further data are added to the chart consideration should be made for shifting the average and control limits per guidance below.

4.1 Individual Point(s) Outside the Control Limits (Outliers)

If the initial control chart shows individual point(s) outside of the control limits, consider recalculating both the average and standard deviation (which affects the control limits) without those point(s) included.

Ideally, the cause for the outlier(s) should be determined and verified that each was a special case that will probably not recur.

After recalculating the average and control limits, look at the remaining data. If the new average line appears to split the remaining data better than the original average (equal data points on either side of the average), then this confirms the credibility of removing the outliers).

After recalculating the average and control limits, further points may have become outliers. In severe cases, this can lead to an endless series of throwing out "outliers" until very little data are left. In such cases, it is best to revert back to the original average and control limits.

4.2 Individual Point(s) Close to the Control Limits

In some cases, individual point(s) may be so close to the control limits that if they were removed from the average and control limit calculations, these point(s) would become outside the control limits.

This is a "gray" area. Confirm that this action should be taken prior to taking it. Confirmation includes either:

- Knowing (from knowledge of the process) that it is a special case and knowing its cause, or

- Review of the average line shows that it is skewed toward the potential outlier when compared with the remaining data.

4.3 Multiple Points Criteria Above/Below Average

This condition indicates that there is not a single, stable baseline. The data may need to be split into two or more regions. The average and standard deviation (and control limits) need to be calculated for each region.

Ideally, the cause for the change between each region should be determined.

Each region must be non-overlapping. That is, do not calculate one region as January 98 through July 98 and the next region as June 98 through November 98.

Try to line up the breaks between regions with jumps in the data. It is especially good if the first data point of the region is outside the control limits of the previous region.

Try to keep at least 8 data points in each region. If there is less than 8, consider not putting any average or control limit on the short region and/or circling the data points related to the trend.

4.4 <u>Seven Points in a Row all Increasing/Decreasing</u>

This condition indicates that a continuous change (ramp) is occurring in the data. There are 2 schools of thought for these periods:

- Do not add any average or control limits, as the process is not stable. Statistical Process Control is meaningless in this condition.

- Add average and control limits. The Statistical Process Control (SPC) technique verifies that the data are undergoing continuous, significant change. Statistical Process Control can be thought of as a formal "test" for the existence of significant change(s). In this case, the "test" shows that significant change is occurring.

If average and control limits are added, show the continuous change as a series of step changes using average and control limits over multiple, non-overlapping regions. Considerations are similar to the methods for seven points in a row above/below average.

If the regions become too short, simply leave one longer time interval region with points going from below the LCL to above the UCL (or vice-versa). Again, the SPC can be thought of as "testing" for the existence of the trend.

4.5 <u>Effects on the Standard Deviation</u>

Following removal of outliers or breaking the data into two or more regions, and recalculating a new standard deviation, there should be a decrease in the standard deviation. This will cause the control limits to be closer to the average. If there is not an obvious reduction in the standard deviation, splitting the data or throwing out the outlier may not be justified.

4.6 <u>Actions Following Significant Change in an Existing Performance Indicator</u>

For a new datum outside of the control limits (single datum point significant shift)

- Determine cause.

- Determine actions (Corrective or Reinforcing, depending on direction of change) to be taken. "No action required" is also acceptable, if this is a conscious decision.

- Continue to monitor the PIs.

If future data returns to within the control limits, leave existing average and control limits in place. This was a special case.

If future data remains outside (or close to) the control limits, consider calculating new average and control limits. Follow suggestions for seven points in a row case (below).

For seven points in a row all above average/below average/increasing/decreasing, or other multiple point trend:

- Determine cause.

- Determine actions (Corrective or Reinforcing, depending on direction of change) to be taken. "No action required" is also acceptable, if this is a conscious decision.

- Flag (label) the significant change on the graph.

- Consider calculating new average and control limits. It may be a good idea to wait for additional data and use at least 8 data values to make the new average and control limits. If there are less than 8 data values, the data should be visibly identified as being "stable." This is a subjective decision on the part of the analyst.

- Overall, the considerations are the same as evaluating an initial baseline.

If there is a decision to calculate new average and control limits, it will generally be from much less than 25 data points. Continue to monitor incoming data. It may be necessary, after a few more data points, to recalculate the average and control limits by incorporating new data. Only recalculate with new data if it is apparent that this action is needed. Do not recalculate the average and control limits by incorporating each new datum every month or at the end of a calendar year or fiscal year.

5.0 MAINTAINING A CONTROL CHART

5.1 Each New Time Period

Extend the average line horizontally for x- and c-charts. Also extend the control limit lines horizontally. For p- and u-charts calculate the control limits based upon current baseline average.

Early data may be dropped if the graph becomes too compressed. However, it is best to keep at least 25 points on the graph.

Document the time interval for which the average and control limits were calculated.

Do not recalculate the average each time interval for any of the control charts, and for x- and c-charts do not recalculate the control limits each time interval.

5.2 Examine the Graph Each Time Period for Significant Changes (Trends)

See criteria in item 3.7

5.3 Once a Statistically Significant Change Occurs

A statistically significant change should be noted on the graph by circling the change. Once the data steadies out at a new level, calculate a new average and control limits according to Section 4.

6.0 CONTROL CHARTS RESULTS

Use of Control Chart Results

Control charts serve an important function. The control charts include methodology to detect significant change. The charts include a baseline average line, and control limits. The control limits act as alarm values. If a performance measure datum point falls outside of the control limits (or other significant changes are detected using the other criteria listed above), then this is a significant event, which demands investigation and action. To reinforce this idea, it is useful to document the "owner" of each performance measure and what management decisions have been made or will be made based upon this measure.

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Author:	Steven S Prevette 509-373-9371
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This document is also contained in Appendix 4 of the Project Hanford Management Contract - ESH&Q Performance Indicator Plan

Example Control Charts

Below are links to four example control charts, and text related to their construction and interpretation. Each of these files is approximately 10 kilobytes in size.

- Example X-chart : Work Package Processing Time
- Example P-chart : Percent of Reports due to Design Problem
- Example C-chart : Event Reports per Month
- Example U-chart : Cases per 200,000 hours

Example Control Chart Analyses

For comments or suggestions on this Page, feel free to send them via [email](#) to Steve Prevette or call at 509-373-9371.
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EXAMPLE X-CHART

The X-chart

The chart below is one example of the use of this generic chart. The processing times for the first 39 work packages written in 1997 have been gathered and plotted.

An initial average was calculated for all 39 packages, but it revealed that the most recent seven packages in a row were above average. Therefore, the baseline average was cut back to packages 1 through 32. Packages 33 through 39 are annotated as seven months in a row above average.

Note that the average line is annotated with the basis for the average. The control limits were set at three standard deviations above and below the average line. Packages 1 through 32 were also used in the standard deviation calculation.

The x-axis is annotated as being the work package number. It is assumed that this is a time sequence. The next package written will be 97-0039. One warning on cycle time graphs - only plot the cycle times for those items which have completed the cycle. If you plot items which are still within the cycle, their times will be artificially low.
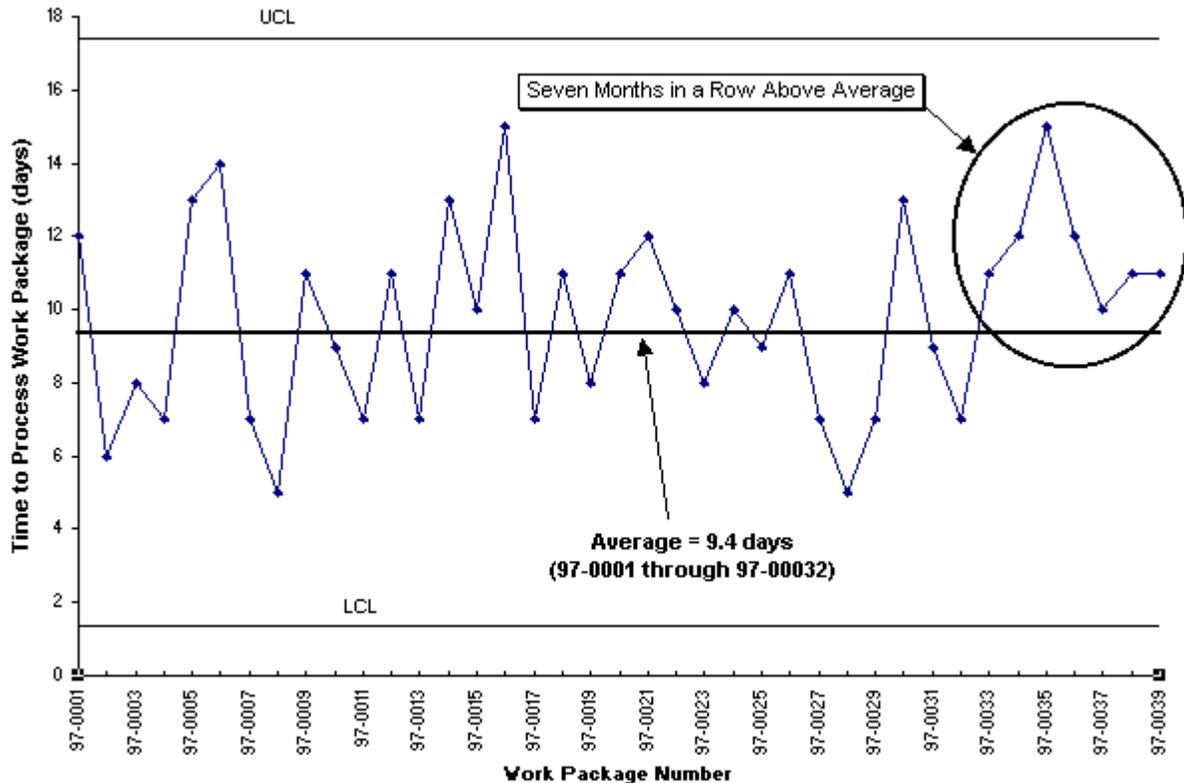
Future Actions: A decision will need to be made as to whether to create a new baseline average for packages 97-0033 through 97-00??, or whether to keep the current average. This decision will depend on if future data returns to the current average or not.
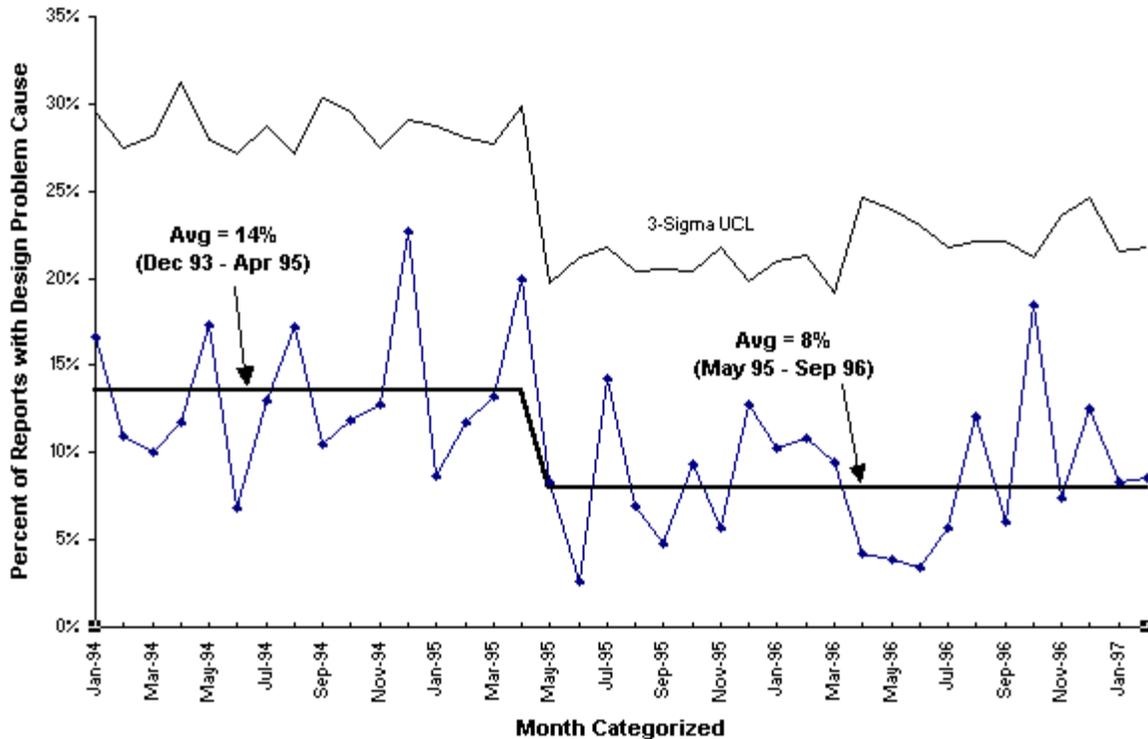
EXAMPLE P-CHART

The P-chart

The chart below is an example of a P-chart. It is graphing the percent of problem reports which were caused by Design Problem. This data is valid for a p-chart as repeated go - no go trials are being conducted; each report was either caused by Design Problem, or was caused by a different cause.

There are two baseline time periods on the graph: December 1993 through April 1995 and May 1995 through September 1996. Note that the current baseline average ends with September 1996. That was when the average was established. New data points have been added each month to this graph, and compared to the 8% baseline. The average has not been (and should not be) recalculated to include each new datum point. The average will only be recalculated once a statistically significant change is detected.

The control limits vary based upon the total number of problem reports each month. The more reports, the less variation expected, and the control limit comes in closer to the baseline average. In this case, the lower control limit was less than zero, so is not shown.

EXAMPLE C-CHART

The C-chart

The chart below is an example of a C-chart. It is counting the number of occurrence reports per month. Since there are multiple dates on Department of Energy occurrence reports (discovery date, categorization date, final report date), the date used for the basis of the graph is identified on the x-axis label.

The c-chart is used when counting discrete events, as in a random arrival process. Counting events is a good example of a random arrival process, as long as each of those events are independent from each other, and overall occur at a constant rate. This is also known as a Poisson process.

In this graph, an initial 24 month baseline was established at the end of 1994 for January 1993 through December 1994. This baseline remained valid until the end of 1995. Then a shift occurred, with 10 of the next 11 points below average (starting with November 1995). A new baseline was established for November 1995 through September 1996. Note the gap in the two baselines from January 1995 through October 1995. This is acceptable. One does want to avoid overlaps between adjoining baseline averages, however.
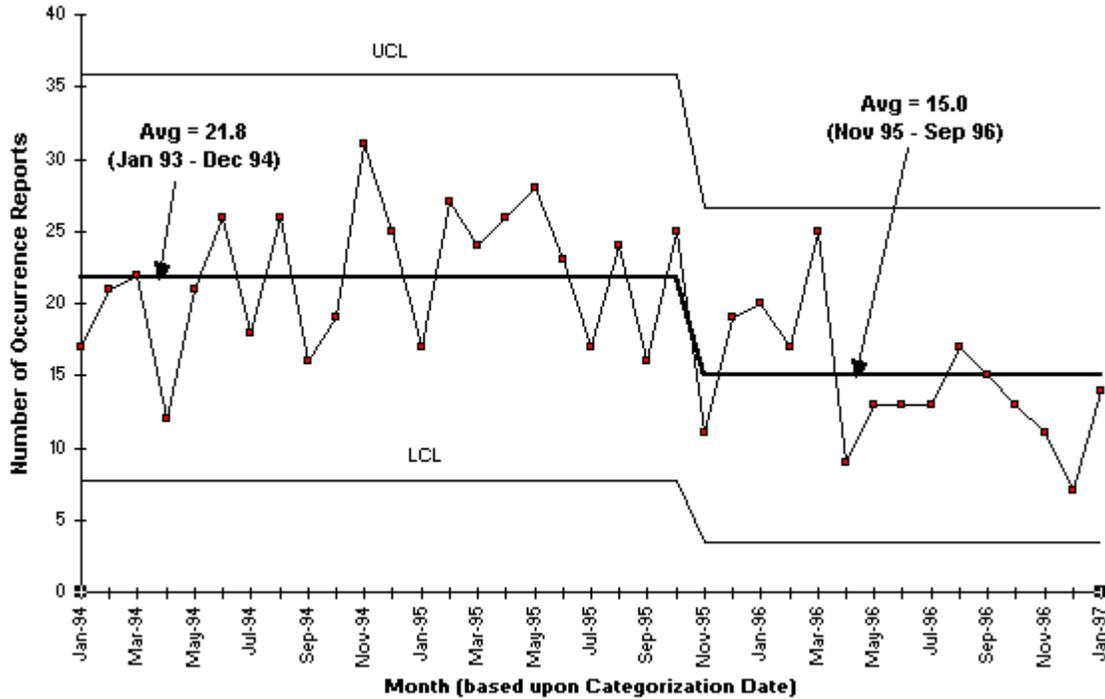
The current baseline (Nov 95 - Sep 96) was based on much less than 25 points. It does appear that the new data is coming in lower than the new baseline (but so far, no statistically significant difference has been detected). If a statistically significant difference does generate, perhaps the choice of November 1995 through September 1996 was insufficient. A better alternative may turn out to be to continue to 21.8 average through March 1996, and calculate a new baseline starting in April 1996. This serves to illustrate how control charts can evolve over time, and baselines with less than 25 points may prove to need to be readjusted.

EXAMPLE U-CHART

The U-chart

The chart below is a typical chart used by a safety department - cases per 200,000 hours. Rather than just plotting the number of accidents, this graph plots accidents per 200,000 hours. This allows the rate on the graph to be consistent even with different size work forces.

In this example, the number of cases each month is determined, then divided by the number of hours worked in the month, and finally multiplied by 200,000. The control limits vary from month to month as hours change. When hours worked are low, the control limits are far from the average line. One expects a large amount of variability in the data when there is a small work force. When hours worked are high, the control limits move inward. One expects a small amount of variability.

Note that the initial average was calculated for a time period starting prior to the beginning of the graph. This is typical for an existing graph which has accumulated many years of data, and a

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decision is made to remove some of the "old" data. In this case, it was decided to remove the data prior to fiscal year 1995. However, the baseline average was left as is, rather than recalculating it for Oct 94 to Jan 95.

There are several significant trends in this graph. First was a decrease from 2.28 to 1.64 in early 1995. The five points from Feb 95 to Jun 95 were greater than one standard deviation below the previous average. However, there was then a significant spike in August 1995. August 1995 reflects special cause variation, and we will assume the cause in question was identified. When developing the proper average to use, it was decided to remove August 1995 from the average. Note the average line cuts through the center of the remaining data for the time interval.
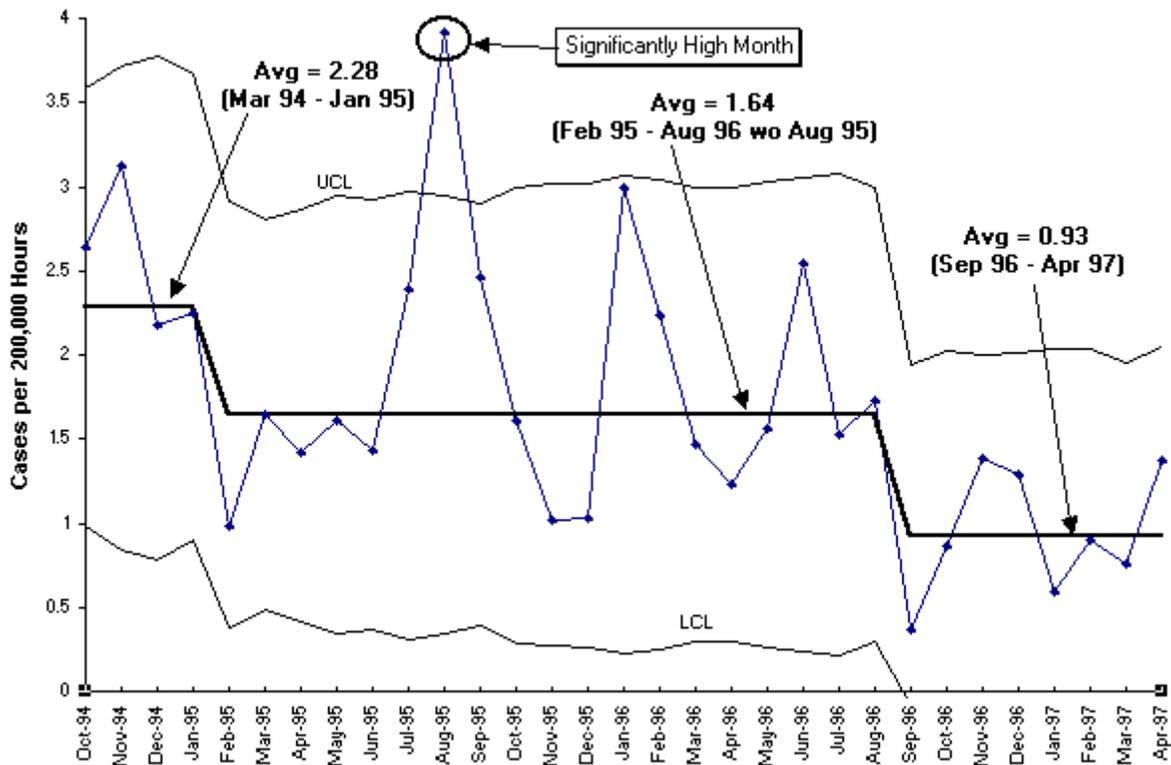
A more recent shift of seven points in a row below average occurred from Sep 96 to Mar 97. As April 1997 remained below the 1.64 previous average rate, an initial baseline was calculated for Sep 96 through Apr 97. During this interval, the Lower Control Limit was mathematically less than zero. Since you cannot have a negative case rate, the Lower Control Limit is not plotted.

This new baseline contains less than 25 points, so it may not be stable. However, this eight point average does provide some basis to determine if a decrease in accidents is continuing, or if the accident rate has steadied out.