Gage Repeatability and Reproducibility

Gage Repeatability and Reproducibility (Gage R&R)

- What is Gage R&R?
  - It is a statistical approach of determining if a gage or a gaging system is suitable for the process under measurement.
- Purpose
  - Measurement is an integral part of any manufacturing unit and is useful in predicting the quality of the manufacturing process.
  - The technique is very useful in predicting the inherent variation in the process, if any, as too much in-process variations can cause serious problems.
- Terms in Gage R&R:
  - Gage:
    - A gage is any device that is used to obtain measurements.
  - Part:
    - A part is an item that is subject to measurement. Typically a part should represent the entire operating range of the process under consideration.
  - Trial:
    - A trial is a set of measurements on a part that is taken by an operator or a computer.
  - Measurement System:
    - A measurement system is the complete process of obtaining measurements. This includes gages, people, procedures and operations.
Types of Measurement System Variation

Bias:

Bias is the difference between the observed average of measurement and the reference value. It is the systematic error that is an indication of a measuring instrument. The reference value is determined by averaging several measurements using standard measuring equipment.

Bias Example

Consider the following ten measurements by an appraiser. The reference value determined by layout inspection equipment is 0.80 mm.

\[
\begin{align*}
X_1 &= 0.75 & X_6 &= 0.80 \\
X_2 &= 0.75 & X_7 &= 0.75 \\
X_3 &= 0.80 & X_8 &= 0.75 \\
X_4 &= 0.80 & X_9 &= 0.75 \\
X_5 &= 0.65 & X_{10} &= 0.70
\end{align*}
\]

The observed average is the sum of the measurements divided by 10.

\[
X = \frac{\sum X}{10} = \frac{7.5}{10} = 0.75
\]

Bias = Observed Average – Reference Value

\[
\text{Bias} = 0.75 - 0.80 = -0.05
\]
Repeatability:

Repeatability is the variation in measurements obtained with one measurement instrument when used several times by an appraiser while measuring the identical characteristic on the same part. It is also commonly known as equipment variation.

In the above figure, the repeatability of Gage A is more than that of Gage B as shown by their probability density functions.

Reproducibility:

Reproducibility is the variation in the average of measurements made by different appraisers using the same instrument when measuring the identical characteristic on the same part. It is commonly known as appraiser variation.
Stability:

Stability is the total variation in the measurements obtained with a measurement system on the same master or parts when measuring a single characteristic over an extended time period. Stability is sometimes referred to as drift.
Linearity:

Linearity is the difference in the bias values through the expected operating range of the gage.

Part Variation:

The Part Variation is essentially a measure of the variation of the process. If a large number of parts made by a process are measured, 99% (5.15 \( \sigma \)) of the parts would be within the variation limits. The Part Variation is always less than or equal to the total variation. In most industrial processes the part variation is large compared to the gauge variation and so the assumption that the observed standard deviation is approximately equal to the total population standard deviation holds good.

Methods to determine Gage R&R

There are three basic and widely used methods for determining the Gage R&R. They are:

- Range method
- Average and Range method
- Analysis of Variance method (ANOVA)

Let us see the Average and Range method in detail.

The Average and Range method is a statistical method that provides an estimate of the following components:

- Part Variation
- Repeatability
- Reproducibility
- R&R
- Total Variation

This method computes the total measurement system variability, which can be separated into components like repeatability, reproducibility and part variation.
The Average and Range method requires multiple parts, appraisers and trials to quantify the repeatability and reproducibility. The following is a typical Data sheet used in industries.

### Average and Range Method (Example)

Consider the following example: (Taken from Measurement System Analysis Reference Manual)

<table>
<thead>
<tr>
<th>Appraiser</th>
<th>Trial #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>Average</th>
<th>Range</th>
<th>Target</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>217</td>
<td>220</td>
<td>217</td>
<td>214</td>
<td>216</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>216.8</td>
<td></td>
<td></td>
<td></td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>216</td>
<td>216</td>
<td>216</td>
<td>213</td>
<td>219</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>215.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>216</td>
<td>216</td>
<td>216</td>
<td>212</td>
<td>219</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>216.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>216.3</td>
<td>216.3</td>
<td>216.7</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td>216.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>219</td>
<td>216</td>
<td>215</td>
<td>212</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>218.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>220</td>
<td>220</td>
<td>216</td>
<td>212</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>217.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td>218.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No. of Appraisers = 2  
No. of Trials = 3  
No. of parts = 5  

\[
\bar{X} = \frac{(X_1 + X_2)}{5}
\]

\[
S_{app} = N_{app}(\bar{X}) - \bar{X}
\]

\[
LCL_u = \bar{X} - 3 \cdot S_{app}
\]

\[
UCL_u = \bar{X} + 3 \cdot S_{app}
\]

\[
LCL_l = \bar{X} - 3 \cdot (\bar{X} - X_{min})
\]

\[
UCL_l = \bar{X} + 3 \cdot (\bar{X} - X_{max})
\]

\[
LCL_R = \bar{R} - 3 \cdot (\bar{R} - R_{min})
\]

\[
UCL_R = \bar{R} + 3 \cdot (\bar{R} - R_{max})
\]

\[
\bar{R} = \frac{(R_1 + R_2)}{5}
\]

\[
S_{app} = N_{app}(\bar{R}) - \bar{R}
\]
Repeatability – Equipment Variation (EV):

\[ EV = \bar{R} \times K_1 \]

\[ = 2.5 \times 3.00 \]

\[ = 7.5 \]

Note:

• All calculations are based upon predicting 5.15σ (99% area under the normal curve)

\[ K_1 = \frac{5.15}{d_2} \]

where \( d_2 \) depends on the no. of trials (m) and the number of parts times the no. of appraisers (g). The value of \( d_2 \) is obtained from Table 1.

In our case \( m = 3 \) and \( g = 2 \times 5 = 10 \). Looking up Table 1 we get \( d_2 = 1.72 \).

Therefore \( K_1 = \frac{5.15}{1.72} = 3.00 \).

Reproducibility – Appraiser Variation (AV):

\[ AV = \sqrt{\left( \frac{1}{(n - 1)} \times \frac{1}{r} \times d_2 \times K_2 \right)^2} \]

\[ = \sqrt{\left( \frac{1}{(n - 1)} \times \frac{1}{r} \times d_2 \times K_2 \right)^2} \]

\[ = 1.0461 \]

Note:

• If a negative value is calculated under the square root sign, the value AV defaults to zero.

\[ n = \text{No. of parts} \]

\[ r = \text{No. of Trials} \]

\[ K_2 = \frac{5.15}{d_2} \]

where \( d_2 \) depends on the no. of appraisers (m) and g is 1, since there is only one range calculation.

In our case \( m = 2 \). Looking up Table 1 we get \( d_2 = 1.41 \)

Therefore \( K_2 = \frac{5.15}{1.41} = 3.65 \).

All the points are within limits (UCL and LCL) and so the measurement process is under control and is said to be consistent.
Repeatability and Reproducibility (R&R):

\[ R & R = \sqrt{EV^2 + AV^2} \]
\[ = \sqrt{7.5^2 + 1.0461^2} \]
\[ = 7.57 \]

Part Variation (PV):

\[ PV = R_p \times K_3 \]
\[ = 6.15 \times 2.08 \]
\[ = 12.79 \]

Note:

- \( K_3 = \frac{5.15}{d_2} \) where \( d_2 \) is dependent on the no.of parts (m) and g = 1, since there is only one range calculation.
- In our case m = 5 and g = 1. Looking up Table 1 we get \( d_2 = 2.48 \).
- Therefore \( K_3 = \frac{5.15}{2.48} = 2.08 \).

There are three points that fall outside the limits and so the measurement process is not adequate to detect part-to-part variations.

\[ UCL_X = \overline{X} + A_2 \overline{R} \quad LCL_X = \overline{X} - A_2 \overline{R} \]

For constant \( A_2 \) look up Table 2.
Total Variation (TV):

\[ TV = \sqrt{(R & R)^2 + (PV)^2} \]

\[ = \sqrt{7.57^2 + 12.79^2} \]

\[ = 14.86 \]

Summary:

<table>
<thead>
<tr>
<th>Study Date</th>
<th>Type</th>
<th>Name</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM-09-YY</td>
<td>xxxx</td>
<td>xxxx</td>
<td></td>
</tr>
</tbody>
</table>

Table 1

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.41</td>
<td>1.91</td>
<td>2.24</td>
<td>2.64</td>
<td>2.67</td>
<td>2.83</td>
<td>2.96</td>
<td>3.05</td>
<td>3.18</td>
<td>3.27</td>
<td>3.35</td>
<td>3.42</td>
<td>3.49</td>
</tr>
<tr>
<td>2</td>
<td>1.28</td>
<td>1.81</td>
<td>2.15</td>
<td>2.40</td>
<td>2.65</td>
<td>2.77</td>
<td>2.91</td>
<td>3.02</td>
<td>3.13</td>
<td>3.23</td>
<td>3.30</td>
<td>3.38</td>
<td>3.45</td>
</tr>
<tr>
<td>3</td>
<td>1.23</td>
<td>1.77</td>
<td>2.12</td>
<td>2.38</td>
<td>2.65</td>
<td>2.75</td>
<td>2.95</td>
<td>3.01</td>
<td>3.11</td>
<td>3.22</td>
<td>3.29</td>
<td>3.37</td>
<td>3.47</td>
</tr>
<tr>
<td>4</td>
<td>1.21</td>
<td>1.75</td>
<td>2.12</td>
<td>2.38</td>
<td>2.65</td>
<td>2.75</td>
<td>2.95</td>
<td>3.01</td>
<td>3.11</td>
<td>3.22</td>
<td>3.29</td>
<td>3.37</td>
<td>3.47</td>
</tr>
<tr>
<td>5</td>
<td>1.19</td>
<td>1.74</td>
<td>2.12</td>
<td>2.38</td>
<td>2.65</td>
<td>2.75</td>
<td>2.95</td>
<td>3.01</td>
<td>3.11</td>
<td>3.22</td>
<td>3.29</td>
<td>3.37</td>
<td>3.47</td>
</tr>
<tr>
<td>6</td>
<td>1.18</td>
<td>1.74</td>
<td>2.10</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>7</td>
<td>1.17</td>
<td>1.74</td>
<td>2.10</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>g 8</td>
<td>1.17</td>
<td>1.74</td>
<td>2.10</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>9</td>
<td>1.16</td>
<td>1.74</td>
<td>2.09</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>10</td>
<td>1.16</td>
<td>1.74</td>
<td>2.09</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>11</td>
<td>1.16</td>
<td>1.74</td>
<td>2.09</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>12</td>
<td>1.15</td>
<td>1.74</td>
<td>2.08</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>13</td>
<td>1.15</td>
<td>1.74</td>
<td>2.08</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>14</td>
<td>1.15</td>
<td>1.74</td>
<td>2.08</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
<tr>
<td>15</td>
<td>1.15</td>
<td>1.74</td>
<td>2.08</td>
<td>2.36</td>
<td>2.63</td>
<td>2.73</td>
<td>2.93</td>
<td>3.00</td>
<td>3.10</td>
<td>3.18</td>
<td>3.28</td>
<td>3.36</td>
<td>3.45</td>
</tr>
</tbody>
</table>

Table 2: \( d_1 \), Values for the Distribution of the Average Range

\( (d_1 \) values for \( g > 15 \)
Table 2. Control Chart constants

<table>
<thead>
<tr>
<th>Number of Observations in Subgroup</th>
<th>$A_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.880</td>
<td>0</td>
<td>3.267</td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td>0</td>
<td>2.575</td>
</tr>
<tr>
<td>4</td>
<td>0.729</td>
<td>0</td>
<td>2.282</td>
</tr>
<tr>
<td>5</td>
<td>0.577</td>
<td>0</td>
<td>2.115</td>
</tr>
<tr>
<td>6</td>
<td>0.483</td>
<td>0</td>
<td>2.004</td>
</tr>
<tr>
<td>7</td>
<td>0.419</td>
<td>0.076</td>
<td>1.924</td>
</tr>
<tr>
<td>8</td>
<td>0.373</td>
<td>0.136</td>
<td>1.864</td>
</tr>
<tr>
<td>9</td>
<td>0.337</td>
<td>0.184</td>
<td>1.816</td>
</tr>
<tr>
<td>10</td>
<td>0.308</td>
<td>0.223</td>
<td>1.777</td>
</tr>
<tr>
<td>11</td>
<td>0.285</td>
<td>0.256</td>
<td>1.744</td>
</tr>
<tr>
<td>12</td>
<td>0.266</td>
<td>0.284</td>
<td>1.716</td>
</tr>
<tr>
<td>13</td>
<td>0.249</td>
<td>0.308</td>
<td>1.692</td>
</tr>
<tr>
<td>14</td>
<td>0.235</td>
<td>0.329</td>
<td>1.671</td>
</tr>
<tr>
<td>15</td>
<td>0.223</td>
<td>0.348</td>
<td>1.652</td>
</tr>
</tbody>
</table>