Evaluating The Usefulness Of Data By Gage Repeatability And Reproducibility

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Abstract

The use of Gage Repeatability and Reproducibility (Gage R&R) studies is widespread in industry. Such analyses allow one to estimate the contribution of variation attributable to the measurement system itself. If these estimates indicate that the recorded measurements may be unreliable, this may impact all subsequent analyses, e.g. control charts, capability analyses, etc. It is the aim of this paper to address such issues by the use of discussion and an example, and to provide some useful guidelines and insights when using MINITAB®.

Gage R&R

We shall consider a measurement process whereby several operators use a particular gage. As such, we may consider the following:

1. An effect due to the operator (*Operator*)
2. An effect due to the particular part being measured (*Part*)
3. An operator by part interaction effect (*Op*Part*)
4. The precision of the gage (*Replication*)

The elements that contribute to the reproducibility piece of "R&R" are the *Operator* and *Op*Part* effects. The two-way random effects ANOVA model that will be considered for the purposes of such an analysis may take the form:

$Y_{ijk} = \mu + Operator_i + Part_j + (Op*Part)_{ij} + Replication_{k(ij)},\ i = 1,2,\ldots,a;\ j = 1,2,\ldots,b;\ k = 1,2,\ldots,n$

and the variance components may be represented by the identity:

$\sigma^2_y = \sigma^2_{Operator} + \sigma^2_{Part} + \sigma^2_{Op*Part} + \sigma^2$

The operators and parts are considered to be random factors. Certain practitioners choose some parts for the study that fall in the extremes of recorded measurements, possibly including some outside of the specification limits, in order to obtain a better representation of the overall performance of the measuring system.

Example

Consider a manufacturer of fuel injector nozzles who is required to assess a measurement system with an allowable tolerance of 8 microns. It is decided upon to obtain nine nozzles, measured twice by two operators. It is important to randomize the order in which the operators measure the parts each time. As is discussed by Montgomery and Runger[1] (1993), one would be advised in practice to perform fewer replications on more parts than vice-versa. In the case of destructive testing, one would use the Nested Gage R&R functionality in MINITAB Release 13.

As is shown in Figure 1, use is made of the Gage R&R Study (Crossed) since each operator measures each part. With the ANOVA output corresponding to the full model, we are unable to reject the null hypothesis that the operator by part interaction effect is equal to zero, even at the $\alpha = 0.1$ level. By default, if the p-value for this effect is greater than 0.25, MINITAB will include this term into the error, and repeat the ANOVA computations.

Figure 1

Two-Way ANOVA Table With Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	769.148	0.00000
Oper	1	0.0400	0.04000	5.333	0.04974
Oper*Nozzle	8	0.0600	0.00750	0.675	0.70735
Repeatability	18	0.2000	0.01111		
Total	35	46.4489			

Two-Way ANOVA Table Without Interaction

Source	DF	SS	MS	F	P
Nozzle	8	46.1489	5.76861	576.861	0.00000
Oper	1	0.0400	0.04000	4.000	0.05605
Repeatability	26	0.2600	0.01000		
Total	35	46.4489			

[1] Montgomery, D.C., Runger, G.C. (July 1993). "Gauge Capability and Designed Experiments: Part I: Basic Methods," *Quality Engineering*, Vol. 6, No. 1.

Figure 2

Gage R&R

Source	VarComp	%Contribution (of VarComp)
Total Gage R&R	0.0117	0.80
Repeatability	0.0100	0.69
Reproducibility	0.0017	0.11
Oper	0.0017	0.11
Part-To-Part	1.4397	99.20
Total Variation	1.4513	100.00

Source	StdDev (SD)	Study Var (5.15*SD)	%Study Var (%SV)	%Tolerance (SV/Toler)
Total Gage R&R	0.10801	0.55626	8.97	6.95
Repeatability	0.10000	0.51500	8.30	6.44
Reproducibility	0.04082	0.21025	3.39	2.63
Oper	0.04082	0.21025	3.39	2.63
Part-To-Part	1.19986	6.17925	99.60	77.24
Total Variation	1.20471	6.20424	100.00	77.55

Number of Distinct Categories = 16

In Figure 1 we also find that with the reduced model, the part component is statistically significant, as one would desire, and we are unable to reject the null hypothesis that the operator effect is equal to zero at the 5% level. The variance component computations in Figure 2 indicate that less than 1% of the total variation is due to Gage R&R.

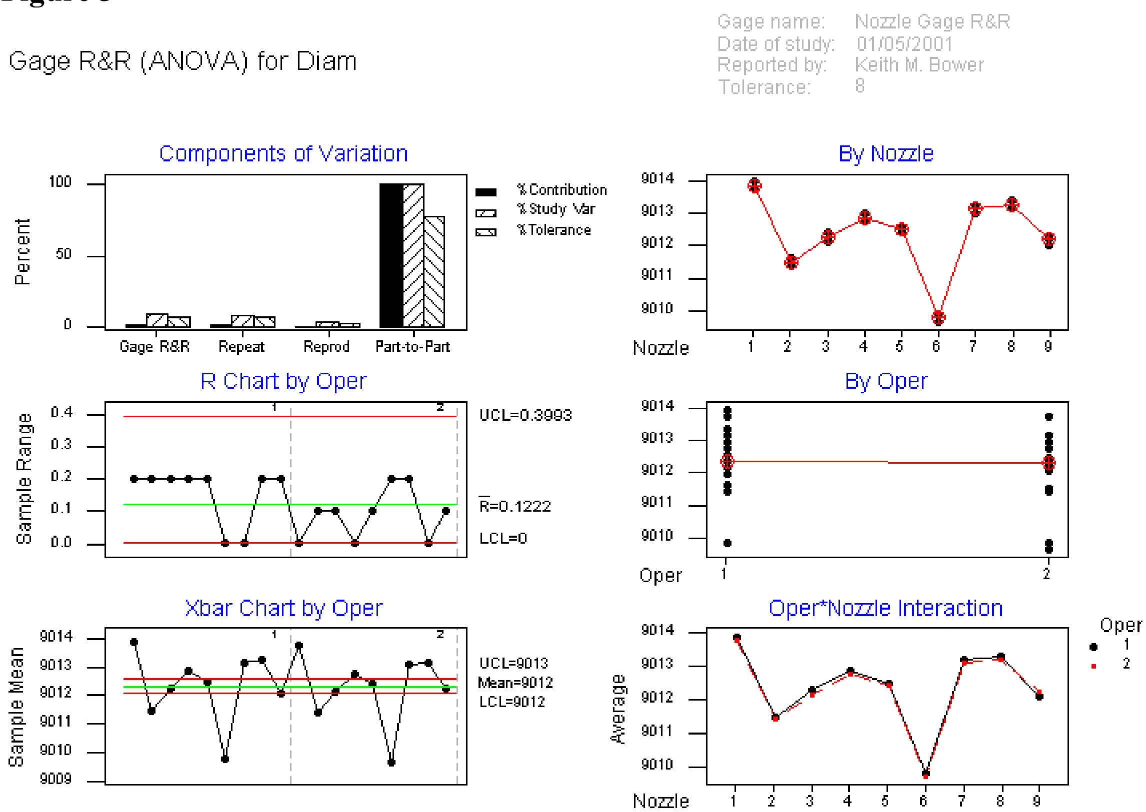
Frequently, practitioners investigate the relationship between allowable tolerances, and/or the Study Variation with the Total Gage R&R computations. As is shown in Figure 2, the process variation used is defined as 5.15 times σ_{total}, where $\sigma_{total} = \sqrt{\sigma^2_{product} + \sigma^2_{gage}}$, hence the estimate of this is used in comparison with 5.15 times the Total Gage R&R (0.55626/6.20424) = 8.97% and with the Tolerance (0.55626/8) = 6.95%. The number of distinct categories indicates how many separate groups of parts the measurement system may be able to distinguish. For example, if the number of distinct categories is two, the process may only distinguish parts by placing into high and low groupings. With 16 distinct categories, the system may be considered very capable of distinguishing between parts. The AIAG[2] states that the "number of categories must be five, and preferably more, for the measurement system to be acceptable…" Under AIAG guidelines, this measurement system would be deemed acceptable.

The graphical output in Figure 3 illustrates how most of the variation is due to the part-to-part component, as one would desire. The R-chart shows that the operators recorded the values for each part with a similar amount of variability, with the Xbar chart indicating an

[2] Automotive Industry Action Group (June 1998). *Measurement Systems Analysis*

out-of-control situation, as one would hope, emphasizing the discriminating power of the instrument. The average values on all parts measured (twice) by the two operators are represented in the "by operator" graph, and indicates that the overall means recorded by both operators are similar. The operator by nozzle interaction effect exhibits parallelism, reflected in the statistically insignificant term being removed from the model.

Figure 3



Conclusion

Obtaining data of high quality is imperative for correct analysis. The use of Gage R&R is a useful component in a measurement system analysis program. Through the use of the ANOVA methodology, along with useful graphs, such insights may be obtained.

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