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GERT ANALYSIS OF DODGE'S CSP-1 CONTINUOUS SAMPLING PLAN*

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SUMMARY. Graphical Evaluation and Review Technique (GERT), originally introduced for systems design and analysis, has been applied here to model and analyse the dynamics of the Dodge's CSP-1 plan. Procedures and tables has been provided to find a unique combination of (i, f) that will achieve the AOQL requirement and also, to optimize the average amount of inspection function, $E(I)$, when the process level $p = p_w$ is known.

1. Introduction

The concept of continuous sampling plan (CSP-1) was introduced by Dodge (1943) as a sampling inspection plan for a product consisting of individual units manufactured in quantity by an essentially continuous process. The detailed procedure and tables for construction and selection of CSP-1 plans have been given by Stephens (1981). Ghosh (1988) and Govindaraju (1989). Whitehouse (1973; 401 - 403) and also Ohta and Kase (1984) and Chakraborty and Rathie (1989) modelled and analysed the Dodge's CSP-1 continuous sampling plan through GERT approach. However, the limitation in the GERT network modeling of the CSP-1 plan in the above studies is two-fold. Firstly, the GERT network representation is limited to the separate modeling; one during detailing state (100% inspection) and, the other during sampling inspection state. Hence they fail to model the dynamics of the plan during one inspection cycle (i.e. 100% inspection to sampling inspection). Secondly, no attention has been paid to the selection of plan parameters in the light of new interpretation of the existing system.

The purpose of the present investigation is to model and analyse the dynamics of CSP-1 plan during one inspection cycle through GERT approach. The advantage of GERT analysis in the present context is two fold. First, this procedure gives the visual picture of the dynamics of the inspection system and

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second, it offers thorough characterization of the plan. Various performance characteristics of interest to quality control engineers and plan engineers have been derived and illustrated numerically. Finally, procedures and tables have been provided for the selection of plans in the context of new developments. This paper presents tables to find a unique combination of plan parameters (i, f) that will achieve AOQL requirement and maximizes the average number of units inspected by CSP-1 plan during one inspection cycle $E(I)$, at $p = p_w$. The $E(I)$ function defined for one inspection cycle is maximum at $p = p_w$ when the system remains in the detailing state for a long time. Thus, $p = p_w$ is the worst incoming quality to be considered by the plan. Therefore, like AOQL the specification of p_w also alarms the state of corrective action to be taken by the producer.

2. Operating procedure of the CSP-1 plan

The operating procedure of the CSP-1 plan as stated by Dodge is as follows :

(a) At the outset , inspect 100% of the units consecutively as produced and continue such inspection until i units in succession are found clear of defects.

(b) When i units in succession are found clear of defects, discontinue 100% inspection, and inspect only a fraction f of the units, selecting individual units one at a time from the flow of product, in such a manner as to ensure an unbiased sample.

(c) If a sample unit is found defective, revert immediately to a 100% inspection of succeeding units and continue until again i units in succession are clear of defects, as in step (a).

(d) Correct or replace with good units, all defective units found.

Thus, the CSP-1 plans are characterised by two parameters i and f.

3. Brief review of GERT

GERT was initiated by Pritsker and Happ (1966), Pritsker and Whitehouse (1966) and Whitehouse and Pritsker (1969) as a procedure for the analysis of stochastic networks having the following features:

(1) Each network consists of logical nodes (or events) and directed branches (or activities).

(2) A branch has a probability that the activity associated with it will be performed.

(3) Other parameters describe the activities represented by the branches. In this paper, however, reference will be made to a sample size parameter only.

The sample size n associated with a branch is characterised by the moment generating function (*mgf*) of the form $M_n(\theta) = \sum_n \exp\,(n\theta) f(n)$, where $f(n)$ denotes the density function of n and θ is any real variable. The probability ϕ that the branch is realised is multiplied by the *mgf* to yield the W-function such that

$$W(\theta) = \phi M_n(\theta) \qquad \ldots (3.1)$$

The W-function is used to obtain the information on the relationship which exists between the nodes.

4. GERT analysis of the plan

The possible states of the CSP-1 inspection system described in section (2) can be defined as follows:

S_0 : Initial state of the plan.

$S_1(k)$: State in which $k(= 1, 2..., i)$ preceding units are found clear of defects during 100% inspection.

SP : Initial state of partial inspection.

S_2 : State in which a unit is not inspected (i.e. passed) during sampling inspection.

SP_A : State in which a unit is found free of defects during partial (sampling) inspection.

SP_R : State in which a unit is found defective during partial inspection.

S_A : State in which current unit is accepted.

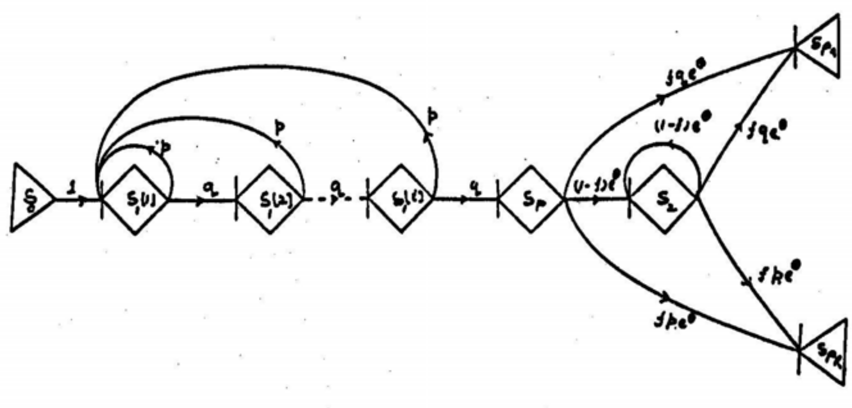
S_R : State in which current unit is rejected.

The above states enable us to construct GERT network representation of the inspection system as shown in Fig. (1) and (2). Suppose that the process is in statistical control, so that the probability of any incoming unit being defective is (p) and the probability of any unit being non-defective is $q = 1 - p$. First of all, we will show that the probability of acceptance and rejection of a unit during sampling inspection [see Fig. (1)] is same as that of its acceptance and rejection during 100% inspection. Now, by applying Mason's (1953) rule in the representation in Fig. (1), the W-functions from the initial node S_0 to the terminal nodes SP_A and SP_R are respectively found as

$$W_{1A}(\theta) = -\frac{fq^{i+1}[1-(1-f)e^{\theta}]e^{\theta} + f(1-f)q^{i+1}e^{2\theta}}{1-[(1-q^i)+(1-f)e^{\theta}]+(1-f)(1-q^i)e^{\theta}} \qquad \ldots(4.1)$$

and

$$W_{1R}(\theta) = -\frac{fpq^{i+1}e^{\theta}[1-(1-f)e^{\theta}] + fpq^{i+1}(1-f)e^{2\theta}}{1-[(1-q^i)+(1-f)e^{\theta}]+(1-f)(1-q^i)e^{\theta}} \qquad \ldots(4.2)$$



Fig(1):GERT network to represent acceptance/rejection during sampling inspection. (CSP-1) Plan

From the W-functions defined above, we obtain the probability that a unit is accepted and rejected respectively by sampling procedure as

$$[W_{1A}(\theta)]_{\theta=0} = q$$

and

$$[W_{1R}(\theta)]_{\theta=0} = p$$

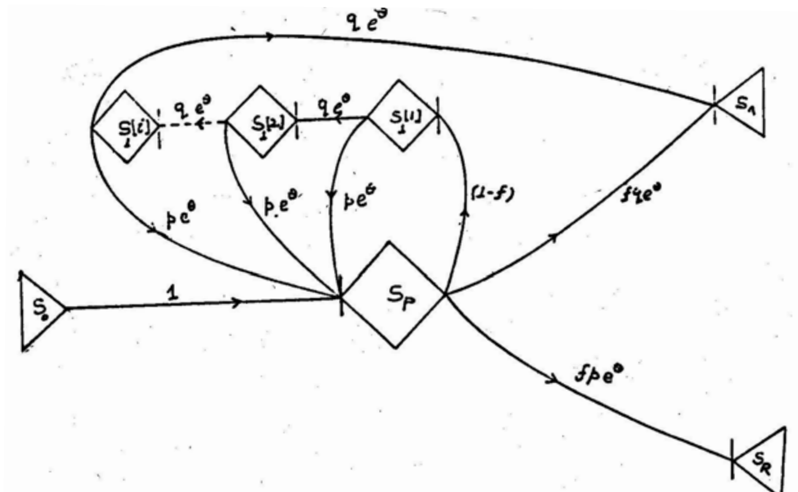
Also, average number of units considered during a period of sampling inspection (v_1) is

$$v_1 = q[\frac{d}{d\theta}M_{1A}(\theta)]_{\theta=0} + p[\frac{d}{d\theta}M_{1R}(\theta)]_{\theta=0} = 1/f$$

where $M_{1A}(\theta) = W_{1A}(\theta)/W_{1A}(0)$ and $M_{1R}(\theta) = W_{1R}(\theta)/W_{1R}(0)$.

Furthermore, Dodge (1943) has shown that average number of units inspected during sampling inspection is $1/p$. Therefore, average number of units passed during sampling inspection is

$$v = (1/f).(1/p) = 1/fp$$



Fig(2):GERT network of CSP-1 inspection system.

Keeping the above fact in mind, the acceptance and rejection sequence of the CSP-1 inspection system during one inspection cycle can be represented by Fig. (2). Consequently, the W-function from the initial node S_0 to the terminal nodes S_A and S_R are respectively given as

$$W_A(\theta) = -\frac{fqe^{\theta} + (1-f)(qe^{\theta})^i}{1-(1-f)pe^{\theta}[\{1-(qe^{\theta})^i\}/(1-qe^{\theta})]} \qquad \ldots(4.3)$$

$$W_R(\theta) = -\frac{fpe^{\theta}}{1-(1-f)pe^{\theta}[\{1-(qe^{\theta})^i\}/(1-qe^{\theta})]} \qquad \ldots(4.4)$$

Therefore,

$$P_A = [W_A(\theta)]_{\theta=0} = [fq + (1-f)q^i]/[f + (1-f)q^i] \qquad \ldots(4.5)$$

and

$$P_R = [W_R(\theta)]_{\theta=0} = fp/[f + (1-f)q^i] \qquad \ldots(4.6)$$

where P_A and P_R stands for probability of acceptance and rejection (of a unit) by CSP-1 plan respectively. These results coincide with Perry (1973) derived for skip-lot sampling plan (SkSP-2).

Since P_A fraction of accepted units are defective with probability p and $(1-P_A)$ fraction are non-defective with probability $q = 1-p$. Again, since all defective units are replaced by good ones, therefore, average outgoing fraction defective (Average Outgoing Quality, AOQ) is defined as

$$\begin{aligned} AOQ &= [pP_A - qP_R]/(P_A + P_R) \\ &= [(1-f)pq^i]/[f+(1-f)q^i] \end{aligned} \qquad \ldots (4.7)$$

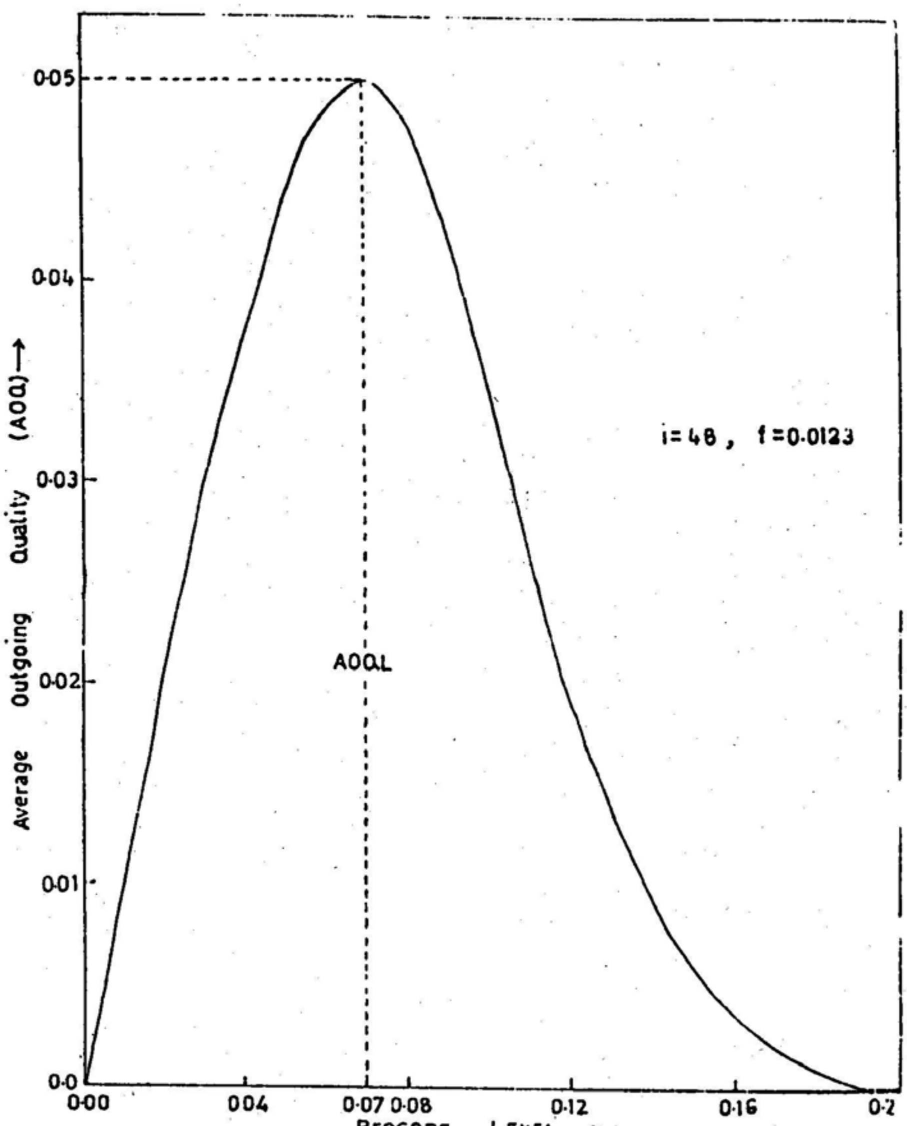


Fig. 3 Average Outgoing Quality Curve for CSP–1 Plans

Proceeding in the same way as above, if defective units found are removed but not replaced, then, average outgoing fraction defective is given as

$$AOQ' = [pP_A - qP_R]/P_A = [(1-f)pq^i]/[fq + (1-f)q^i] \qquad \ldots (4.8)$$

These results agree with Dodge (1943).

For further characterization of the plan, the average number of units inspected by CSP-1 plan during one inspection cycle can be defined as follows:

$$\begin{aligned} E(I) &= P_A[\tfrac{d}{d\theta}M_A(\theta)]_{\theta=0} + P_R[\tfrac{d}{d\theta}M_R(\theta)]_{\theta=0} \\ &= [1 - \{(fq + (1-f)q^i\}]/[fp + (1-f)pq^i] \end{aligned} \qquad \ldots (4.9)$$

where $M_A(\theta) = W_A(\theta)/W_A(0)$ and $M_R(\theta) = W_R(\theta)/W_R(0)$.

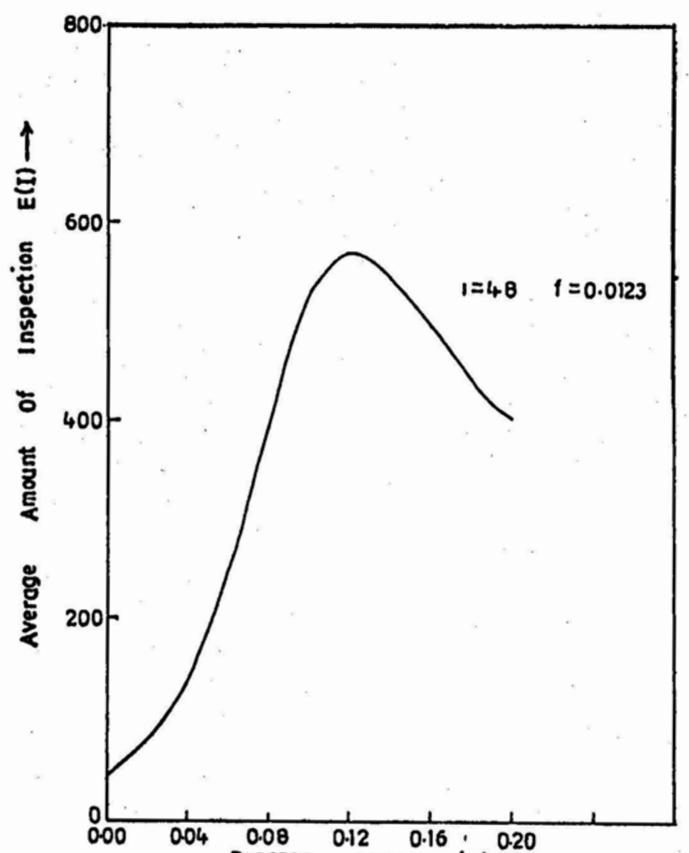


Fig. 4 E(I) curve for CSP-I Plans

Here, it may be observed that when $f = 0$ the resulting plan becomes 100% inspection only. Therefore, putting $f = 0$ in (4.9), *the average amount of inspection*, $E(I)$, comes out to be

$$E(I) = (1 - q^i)/pq^i = u$$

Knowing u and v, the average fraction of units inspected (F) is

$$F = (u + fv)/(u + v) = f/[f + (1 - f)q^i]$$

5. Selection of CSP-1 plans

Dodge (1943) remarked that there are several combinations of plan parameters i and f that will ensure the same AOQL over all possible values of incoming quality p. Stephens (1981) provided tables for selection of (i, f) for consumer protection based on LQL with 0.10 risk. Ghosh (1988) developed a procedure to find a unique (i, f) that will achieve AOQL requirement and minimizes F (average fraction inspected) when the process average $\overline{p}$ is known. Govindaraju (1989) provided tables for the selection of a CSP-1 plan for a given set of conditions (AQL, AOQL) and (LQL, AOQL).

This paper provides procedures and tables to find a unique combination of (i, f) that will achieve AOQL requirement and also, maximizes the average amount of inspection, $E(I)$, when the process level $p = p_w$ is known. This later criterion also alarms the state of corrective action to the producer.

Example. Suppose a CSP - 1 plan is required having AOQL = 0.05 and which maximizes $E(I)$ at $p = p_w = 0.12$. Table (1) yields a CSP-1 plan with $i = 48$ and $f = 0.0123$. Now, for this plan AOQ and $E(I)$ curves have been drawn in Fig. (3) and (4). It is seen from the figure that this plan gives the required AOQL on one hand and also maximizes $E(I)$ at $p = p_w$.

6. Construction of tables

The expression for the average amount of inspection, $E(I)$, is written

$$E(I) = [1 - fq - (1 - f)q^i]/[fp + (1 - f)pq^i]$$

Now, differentiating $E(I)$ with respect to p and equating to zero, one gets

$$f = [ipq^{i-1} - q^i(1 - q^i)]/[(1 - q^i)^2 - ip^2q^{i-1}] \qquad \ldots (6.1)$$

If p_m is the incoming quality at which AOQL occurs, then the following results due to Dodge (1943)

$$f = q_m^{i+1}/[i(\text{AOQL}) + q_m^{i+1}] \qquad \ldots (6.2)$$

where $q_m = 1 - p_m$ and $p_m = [1 + i\ (\text{AOQL})\ ]\ /\ (i + 1)$.

Thus, from equations (6.1) and (6.2), on simplification, we have

$$pq^{i-1}[i^2(\text{AOQL}) + iq_m^{i+1}(1+p)] - (1-q^i)[q_m^{i+1} + i(\text{AOQL})q^i] = 0 \quad \ldots(6.3)$$

For given value of $p = p_w$ and AOQL, equation (6.3) can be solved for i by numerical methods and then the value of f can be found from equation (6.2). Table (1) is constructed for some selected values of AOQL and proces level p_w.

TABLE 1. VALUES OF i AND f OF CSP-1 PLAN FOR GIVEN AOQL AND PROCESS LEVEL P_w

	AOQL = 0.01			AOQL = 0.02	
p_w	i	f	p_w	i	f
0.0200	459	0.0008	0.040	226	0.0008
0.0210	392	0.0018	0.042	193	0.0019
0.0220	338	0.0036	0.044	166	0.0037
0.0230	293	0.0065	0.046	144	0.0068
0.0240	254	0.0101	0.048	125	0.0113
0.0250	221	0.0176	0.050	109	0.0180
0.0260	191	0.0270	0.052	94	0.0273
0.0270	165	0.0404	0.054	82	0.0405
0.0280	140	0.0600	0.056	70	0.0589
0.0290	114	0.0923	0.058	58	0.0865
0.0292	108	0.1022	0.060	44	0.1410
0.0296	91	0.1373	–	–	–
	AOQL = 0.03			AOQL = 0.04	
p_w	i	f	p_w	i	f
0.055	198	0.0001	0.08	109	0.0009
0.060	149	0.0009	0.09	75	0.0054
0.065	115	0.0031	0.10	53	0.0187
0.070	90	0.0083	0.11	37	0.0485
0.075	72	0.0183	0.12	25	0.1103
0.080	56	0.0385	0.13	8	0.4194
0.085	44	0.0653	0.14	6	0.5166
0.090	32	0.1201	0.15	5	0.5498
	AOQL = 0.05			AOQL = 0.06	
p_w	i	f	p_w	i	f
0.11	63	0.0042	0.11	94	0.0002
0.12	48	0.0123	0.12	70	0.0010
0.13	36	0.0285	0.13	55	0.0035
0.14	28	0.0571	0.14	43	0.0091
0.15	20	0.1043	0.15	34	0.0196
0.16	14	0.1825	0.16	27	0.0368
0.17	10	0.2987	0.17	22	0.0627
0.18	7	0.3877	0.18	17	0.1001
0.19	6	0.4371	0.19	14	0.1503
0.20	5	0.4693	0.20	11	0.2120
0.21	5	0.4930	0.21	9	0.2746
0.22	5	0.5116	0.22	7	0.3265
0.23	4	0.5270	0.23	6	0.3657
0.24	4	0.5401	0.24	6	0.3964
0.25	4	0.5515	0.25	5	0.4208

TABLE 1. Continued

	AOQL = 0.07			AOQL = 0.08	
p_w	i	f	p_w	i	f
0.15	48	0.0031	0.15	63	0.0003
0.16	39	0.0074	0.16	51	0.0011
0.17	32	0.0148	0.17	42	0.0029
0.18	26	0.0265	0.18	35	0.0063
0.19	22	0.0434	0.19	29	0.0119
0.20	18	0.0666	0.20	25	0.0205
0.21	15	0.0968	0.21	21	0.0326
0.22	12	0.1337	0.22	18	0.0488
0.23	10	0.1754	0.23	15	0.0693
0.24	9	0.2184	0.24	13	0.0943
0.25	8	0.2586	0.25	11	0.1229
	AOQL = 0.09			AOQL = 0.10	
p_w	i	f	p_w	i	f
0.18	45	0.0012	0.18	56	0.0002
0.19	38	0.0028	0.19	47	0.0005
0.20	32	0.0056	0.20	40	0.0012
0.21	27	0.0101	0.21	34	0.0027
0.22	23	0.0167	0.22	29	0.0051
0.23	20	0.0260	0.23	25	0.0088
0.24	18	0.0380	0.24	22	0.0143
0.25	15	0.0531	0.25	19	0.0215
0.26	13	0.0713	0.26	17	0.0309
0.27	12	0.0924	0.27	15	0.0426
0.28	10	0.1158	0.28	13	0.0566
0.29	9	0.1409	0.29	12	0.0729
0.30	8	0.1661	0.30	10	0.0910

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