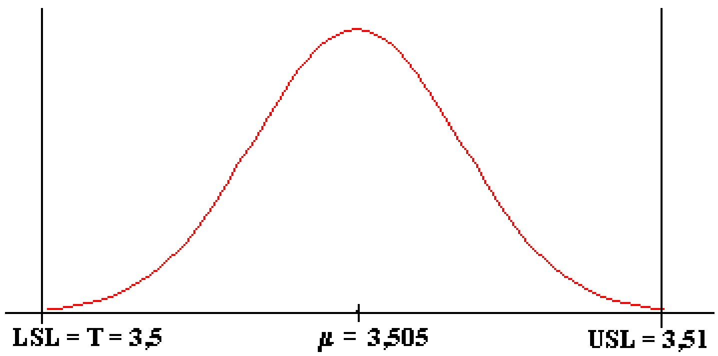
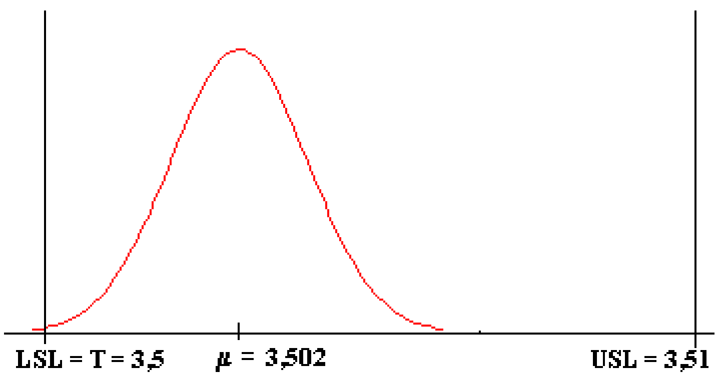
1. **Do you calculate the index Cpk for all types of tolerance?**
   Cpk is inconvenient for a half-tolerance, which is a frequent case.

Problem (K.S.Krishnamoorthi, QE 2(4), 1990):
Let us have a size (T) and tolerance given: $T = 3.5^{+0,01}_{-0,00}$. If we denote the lower tolerance limit as LSL and the upper tolerance limit as USL, then LSL=T=3.5, USL=3.5+0.01=3.51. The figure below depicts two producers. Which of them is better based on a visual assessment of the level of centralization (adherence to T) and of the measure of variability? The first producer's mean is m = 3,505 and his $3s$ variability is 0,005, the second producer's mean is m = 3,502 (thus, closer to T) and his $3s$ variability = 0,002 is smaller:





So, the second producer is clearly better. However, the Cpk index of both of them is the same:

$$Cpk = \min(\frac{USL - m}{3s}, \frac{m - LSL}{3s})$$

a) $Cpk = \min(\frac{3{,}51 - 3{,}505}{0{,}005}, \frac{3{,}505 - 3{,}5}{0{,}005}) = \min(\frac{0{,}005}{0{,}005}, \frac{0{,}005}{0{,}005}) = 1$

b) $Cpk = \min(\frac{3{,}51 - 3{,}502}{0{,}002}, \frac{3{,}502 - 3{,}5}{0{,}002}) = 1$

2. With the use of Cpk, one does not see that the supplier deceives, when he purposefully keeps the mean equal to a tolerance limit:

Why does he do it? For instance, when electronic contacts are guilt, i.e. when a thickness of the gold layer (T) and a tolerance interval (LSL,USL) are set, the mean can be expected to be shifted to the left, as close to the LSL as possible. On the contrary, if the price of a metal plate being sold depends on its weight and the plate thickness (T = 50) is observed, then the average thickness (m = 57) is close to the USL (see the figure 1)

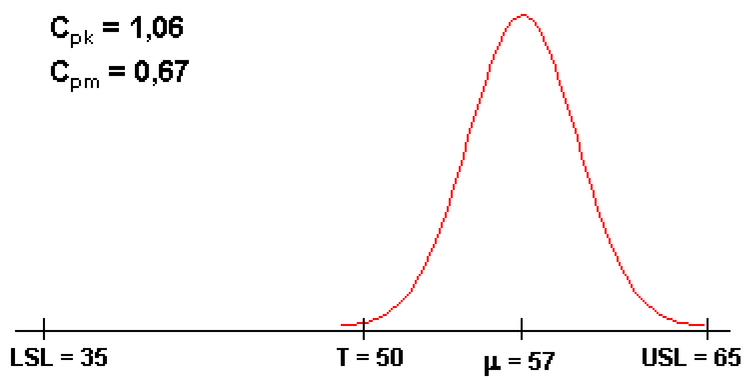


Fig.1 The Cpk is significantly better than Cpm

Why does the Cpk index not reveal this strategy? Not keeping to the target value is offset by reduction in variability:

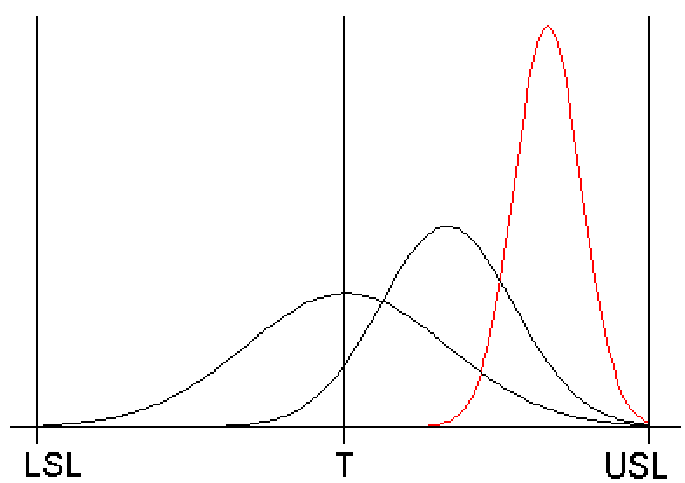


Fig.2 (Boyles, R.A. 1991): Worse centralization is offset by smaller variability. The Cpk = 1 in all the cases

3 Why is it recommended to calculate more capability indices simultaneously?

Two aspects are observed in capability assessment:

a) Ability to keep the target value, which is a so called „process centralization"
b) Measure of variability around T

Why are two aspects observed? The figure 1 shows two producers, both of them are perfectly centralized, but have different variability.

Fig. 1 Different variability

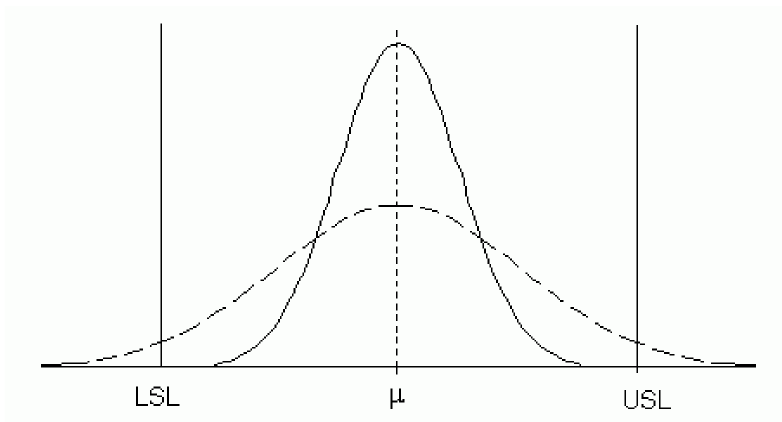
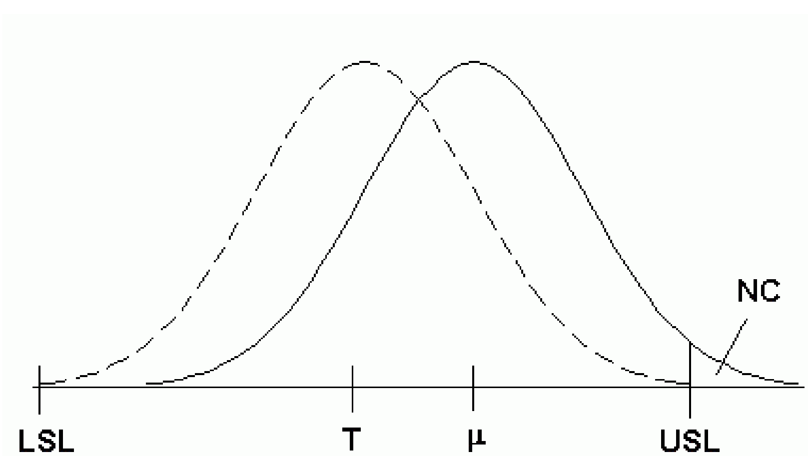


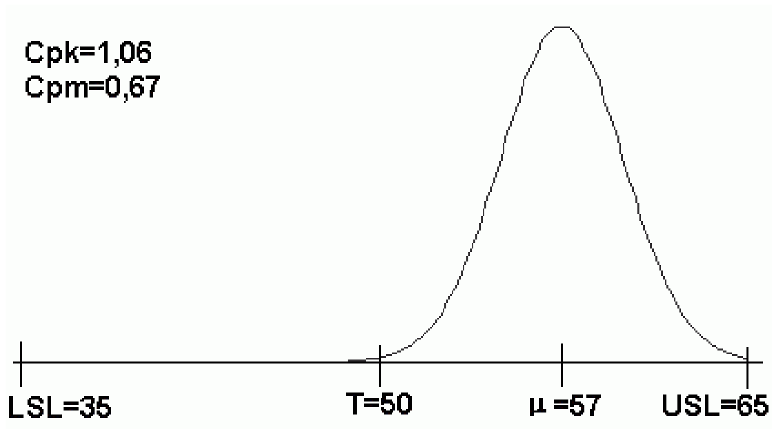
Figure 2 depicts two producers with the same variability, but different level of centralization. It is evident that observance of **one of the characteristics is not enough**.

Fig. 2 Different centralization

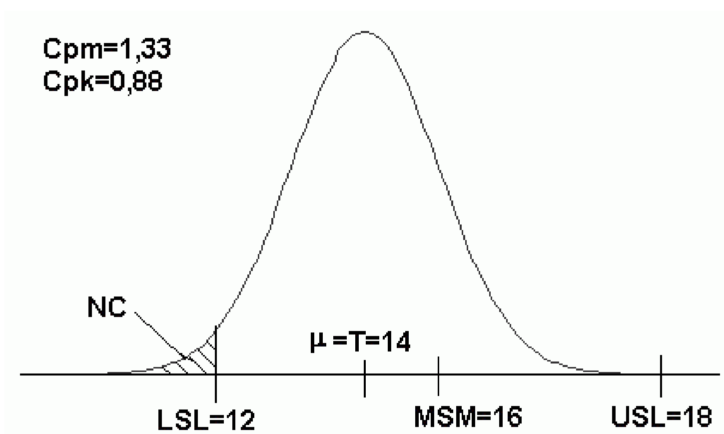


Most indices that are used are always more subject to change when one of the mentioned aspects is not adhered to, as the following problem shows.

Problem: a) Small variability but poor centralization. For the specification of LSL = 35, USL = 65, T = 50, m = 57 and s = 2,5, is Cpk = 1.06 (see the second remark), but Cpm = 0.67. Thus, one index gives a satisfactory result, whereas the other one does not.
The Cpm is changes more when the T value is not met than Cpk is.



b) Good centralization, but high variability. For the specification of LSL = 12, USL = 18, T = 14, m = 14 and s = 0,75 is Cpm = 1.33, but Cpk = 0.88.



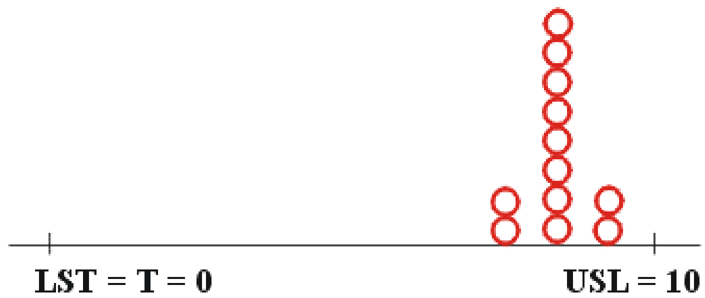
The change of Cpk is greater than that of Cpm when the variability is high.

4. The use of Cpk is not suitable for an S-type tolerance

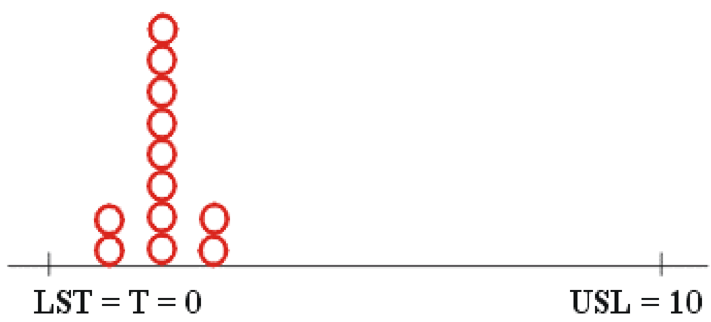
The S-type tolerance has an upper tolerance limit USL set only, the ideal (target) value is T = 0. An example of this use is an allowed amount of injurants in food, a surface roughness or an allowed filter permeability.

(Illustrative) **problem:** T = 0, USL = 10, measurement results:

a) 7,7,8,8,8,8,8,8,8,8,9,9. Cpk = 1.1, but the special index Cs = 0.85.



b) Measurement results: 1,1,2,2,2,2,2,2,2,2,3,3. In this case, the result is closer to the target value, while the variance remains unchanged. The Cpk = 1.1 (no change), but Cs = 3.3.



5. The Cpk index cannot be tested.

The Cpm, on the contrary, can be tested very easily.

Why is it necessary to test the index significance?

If, for example, a supplier requires that Cpm = 1.33, it does not suffice to achieve this value as the calculation was performed using a data sample and the obtained value is thus only an estimate of Cpm. Therefore we ask, what value it is necessary to achieve, i.e. what the Cpm(min) is, so that requested index equals 1.33.

Several values of Cpm(min) are in the following table in case it is demanded that Cpk = 1.33

n	p = 90%	95%	99%
10	Cpm(min)=1,8	2,02	2,50
25	1,61	1,71	1,92
50	1,52	1,58	1,71
75	1,48	1,53	1,63
100	1,46	1,50	1,58

How can the table be interpreted?

If the required value of Cpm is 1.33, then the Cpm(min) depends on:

a) the number of values n from which the index is calculated; the Cpm(min) drops with n growing,
b) the nivel of test p (90%, 95%, 99%).

For n = 10 and the nivel of test p = 90%, for instance, it is necessary to achieve that Cpm(min) = 1.8 so that the demanded value 1.33 of Cpm is met.

A similar table exists for demanded index value of 1.00 and 1.67. If the tables are not available, then the Cpm(min) can be calculated. Another possibility of the index significance verification is testing of its verification. Both the calculation of the Cpm(min) and the index testing can be performed for any n,p and required value of Cpm in the **Capa software.**

6. Capability index gives reliable results
only if the necessary prerequisites are met.

Which prerequisites must be met?
The prerequisites are divided into the *general* ones and *specific* ones:

The *general prerequisites* must be fulfilled for a calculation of every index. They are:

a) Process stability
b) Correct data (independent, representative and without outliers).
c) The right tolerance

Specific prerequisites are related to a specific index, for instance: tolerance symmetry for Cpm, the same value of mean and the target value for Cp, normal distribution for Cpk, Cpm and Cp.

What happens if the prerequisites are not met?

Problem:

a) What an outlier causes:
When a detergent weight is controlled, the following specification is given:
LSL = 19.8, T = 20, USL = 20.2.
Control results: 19.9,20,20,20,20,20,20,20,20,20,20,20.1.
The task is to calculate the index Cpk. In this case, Cpk = 1.56. If there is a mistake in the last number, e.g. when the decimal point was omitted and the calculation used 201 instead of 20.1, then Cpk = 0.00.
Here, the prerequisite (b): correct data-without outliers was not fulfilled.

b) What a faultily set tolerance causes:
If the tolerance interval is wider: LSL = 19 and USL = 21, then even if all the measurement results lie outside the target value, so that none of the values meets the set weight: 19.7, 19.8, 19.8, 19.8, 19.8, 19.8, 19.8, 19.8, 19.8, 19.9, then Cpk = 5.6.
Here, the prerequisite (c): faultily set tolerance is not met.

c) What data autocorrelation causes (Haim Shore, QE 9(4), 1997):
This results in index value reduction, in diversion of the estimate towards higher values and in lower reliability of the index estimate (higher variability of the estimate).

d) *What happens if the prerequisite of normal distribution is not met*
(B.Gunter: Quality Progress, May 1989, p.80):

The figure below depicts a normal distribution denoted as „perfect“ and a mix of normal distribution with 10 per cent of other distribution denoted as „contaminated“. The two distributions do not differ visually too much. Despite that, the perfect distribution Cpk = 1.33, while the contaminated distribution Cpk = 0.50 only. Thus, it can be seen that only a slight diversion from normal distribution causes a significant reduction of the index value.

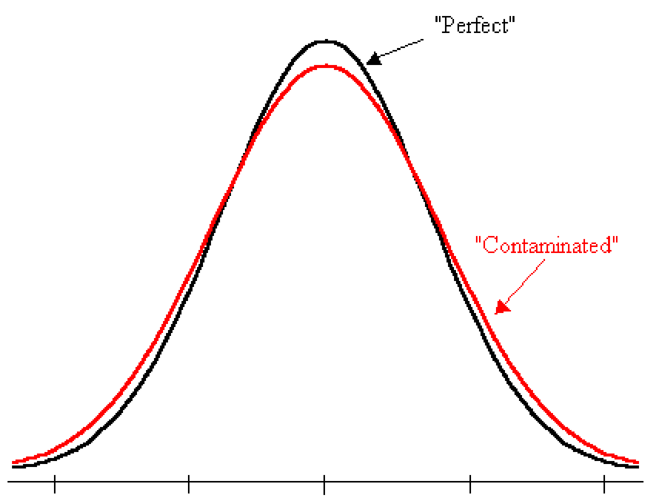


Fig 1: Comparison of Cpk indices when a small diversion from normal distribution occurs.

Fulfilment of general and specific prerequisites is verified by special tests (Capa, the chapter „Tests“).

7. It is unacceptable to use the indices Cp, Cpk, Cpm, Cpmk if the data are not normally distributed.

What happens if the requirement of normality is ignored?

Problem: In a production, the following is set: the target value T = 0.75, lower tolerance limit LSL = 0.68 and the upper tolerance limit USL = 0.82. The results of the control of observed quality characteristics are (illustrative data):0.70, 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78. All the customers are satisfied with the supplied product. However, when the firm manufacturing the product was to prove the process capability using a capability index and it used the Cpk index for this purpose, the Cpk = 0.73. Thus, the process is incapable. We remind that the product brings a general satisfaction. Where is the mistake? The mistake is that the sample is not normally distributed.(Here, it is rather a uniform distribution.)
If the quantiles $x_{0,00135}$ and $x_{0.99865}$ are available, then for a distribution other than normal, it is possible to use the indices Cpp or CpT that were designed by Schneider and his team. The problem with their use is that the quantiles $x_{0,00135}$ and $x_{0.99865}$ can be obtained from the data only if there are at least 740 of them in the sample. But, such a large sample is usually not available. In such a case, the authors recommend to replace the two quantiles with the minimal and maximal value xmin and xmax.

$$Cpp = \min\left\{\frac{\bar{x} - LSL}{\bar{x} - x_{\min}}, \frac{USL - \bar{x}}{x_{\max} - \bar{x}}\right\}$$

$$Cpp = \min\left\{\frac{0.74 - 0.68}{0.74 - 0.70}, \frac{0.82 - 0.74}{0.78 - 0.74}\right\} = \min\{1.5, 2.0\} = 1.5$$

$$C_{pT} = \min\left\{\frac{T - LSL}{T - x_{\min}}, \frac{USL - T}{x_{\max} - T}\right\}$$

$$= \min\left\{\frac{0.75 - 0.68}{0.75 - 0.70}, \frac{0.82 - 0.75}{0.78 - 0.75}\right\} = \min\{1.4, 2.3\} = 1.4$$

The values Cpp = 1.5 or CpT = 1.4 much better reflect the actual level of this technological process.
Since the quantiles were replaced with the minimal and maximal value, it is necessary to have a sufficiently large sample, let us say at least n = 100.

8. How is the capability assessed in case the quality characteristic is not normally distributed?

One of the main criteria determining further steps in the selection of the appropriate capability index is, whether the quality characteristic is normally distributed. Therefore, this prerequisite should be verified before we proceed. The user has two questions to answer:

How to verify normality reliably?

The Capa software contains three tests of normality: the test based on skewness and curtosis, Darling's test and the Shapiro-Wilk's test + QQ graph for graphical decision on normality.

How to assess the capability if the normality is not verified?

In case a test does not confirm accordance with normal distribution, the classical capability indices Cp, Cpk, Cpm, Cpk cannot be used. Basically, there are two other ways of further procedure:

a) Find out what distribution the data have and find the quantiles (for such a distribution) defining the interval which contain 99.73 % of values (generally 100x(1-alpha) percent of values) or use a special capability index defined for this type of distribution. The Capa software contains the generally-known Clement's method for beta and gamma distrubution and t-distribution.

b) Use some of the universal capability indices which do not require normal distribution.

Capa contains
- -the Cpp, CpT and Cs indices
- -confidence interval for Cpk for any type of distribution
- -Castagliola's method for the calculation of Cp and Cpk for any type of distribution

However, even if the normality of the quality characteristic is confirmed, it is not always possible to calculate the basic indices Cp, Cpk, Cpm and Cpmk. Their use is conditioned on the type of tolerance. The indices are not suitable for unilateral tolerances (half-tolerances). Therefore, the Capa has an option for selection of the index that complies with the type of tolerance. The option is in the chapter of N distribution.

9.1 Do you observe more quality characteristics simultaneously?
It is not correct then to assess the capability of each characteristic separately.

How is the capability assessed when more quality characteristics are involved?
The answer is multivariate indices.

When can each quality characteristic be assessed separately? In case they are mutually independent.

Problem: For two variables x1 and x2 (see table 1), the specification is: LSL_1 = 136, T_1 = 176, USL_1 = 216 and LSL_2 = 41, T_2 = 53, USL_2 = 62.5. For x_1 is Cpm = 0.724, for x_2 is Cpm = 0.542, thus incapability in both cases, while the multivariate index MCpm(min) = 0.86, which means that the process is capable.

Tab. 1 Hypothetical data

	X_1	X_2		X_1	X_2
1	143	34,2	13	187	58,2
2	200	57,0	14	186	57,0
3	160	47,5	15	172	49,4
4	181	53,4	16	182	57,2
5	148	47,8	17	177	50,6
6	178	51,5	18	204	55,5
7	162	45,9	19	178	50,9
8	215	59,1	20	196	57,9
9	161	48,8	21	160	45,5
10	141	47,3	22	183	53,9
11	175	57,3	23	179	51,2
12	187	58,5	24	194	57,5
			25	181	55,6

Note: For MCpm, even a value less than one can be sufficient.

9.2 Do you observe more quality characteristics simultaneously?

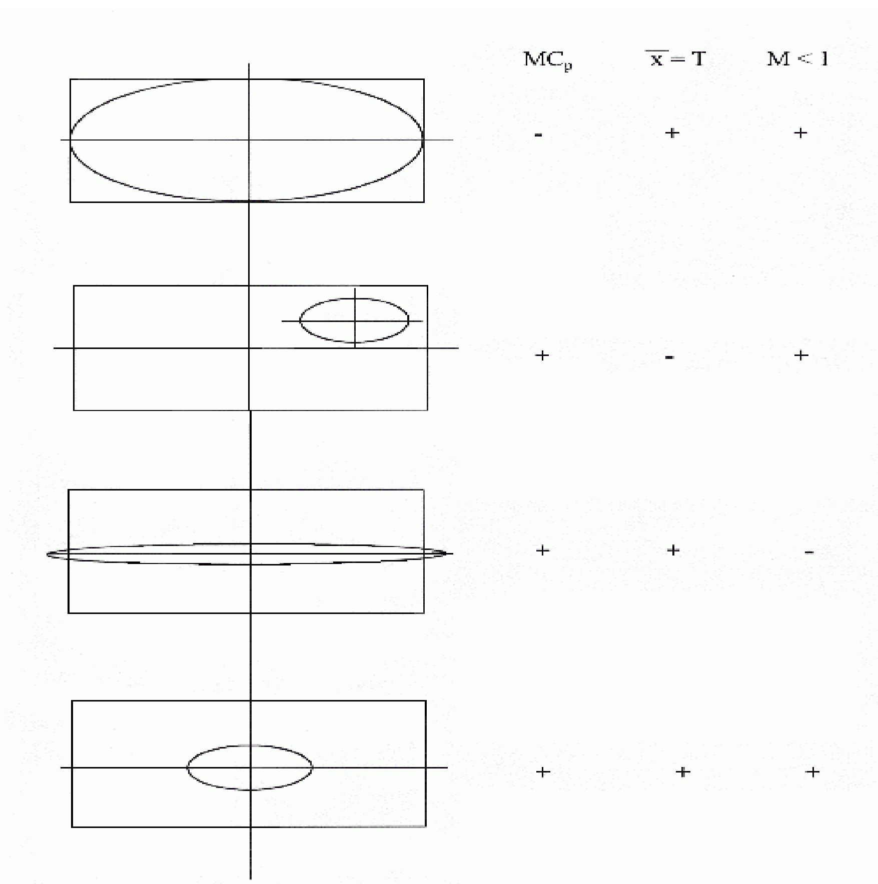
What are the general prerequisites for the capability assessment?

The prerequisites are:

a) Process stability: it is verified by multivariate regulation diagram or by a special test of stability.
b) Correct data: they should be representative, independent (to be verified by a test), without outliers (to be verified by a test as well).
c) Correct tolerance.
d) Normality (to be verified by the test of multivariate normality).

What attributes of multivariate quality characteristics are assessed:

a) Capability (MCp, Mcpm)   b) Process centralization (t-test)   c) Location of measurement results (M statistics)



9.3 Do you observe more quality characteristics simultaneusly?
Is it possible to use a graphical method for a multivariate quality characteristic?

Figure 1 shows a two-dimensional normally distributed quality characteristic, figure 2 depicts a view into the $x_1 0 x_2$ plane limited by tolerance interval for X_1 and X_2 , the target value T_1 and T_2, central point with the coordinates $(\bar{x}_1, \bar{x}_2)$ and the target point $T = (T_1, T_2)$. Assessed are the significance of the distance between the points, variability in the tolerance rectangle defined by the intervals (LSL_1, USL_1) and (LSL_2, USL_2) and the outlying (points outside the tolerance rectangle).

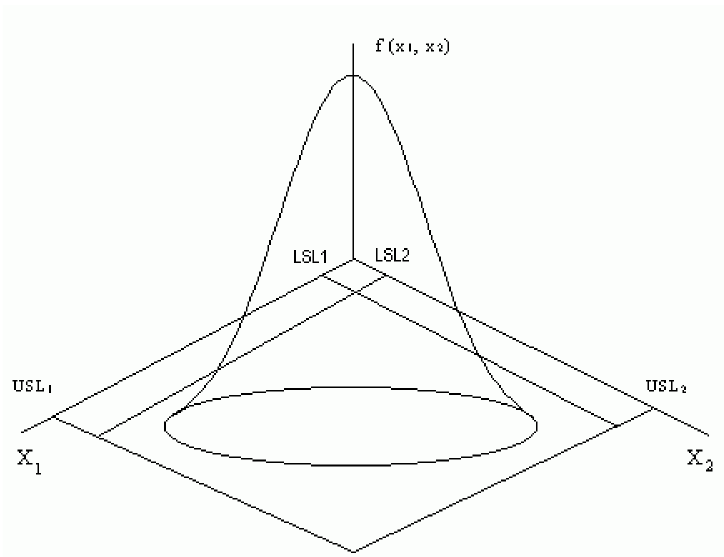


Fig.1 Two-dimensional normally distributed quality characteristic

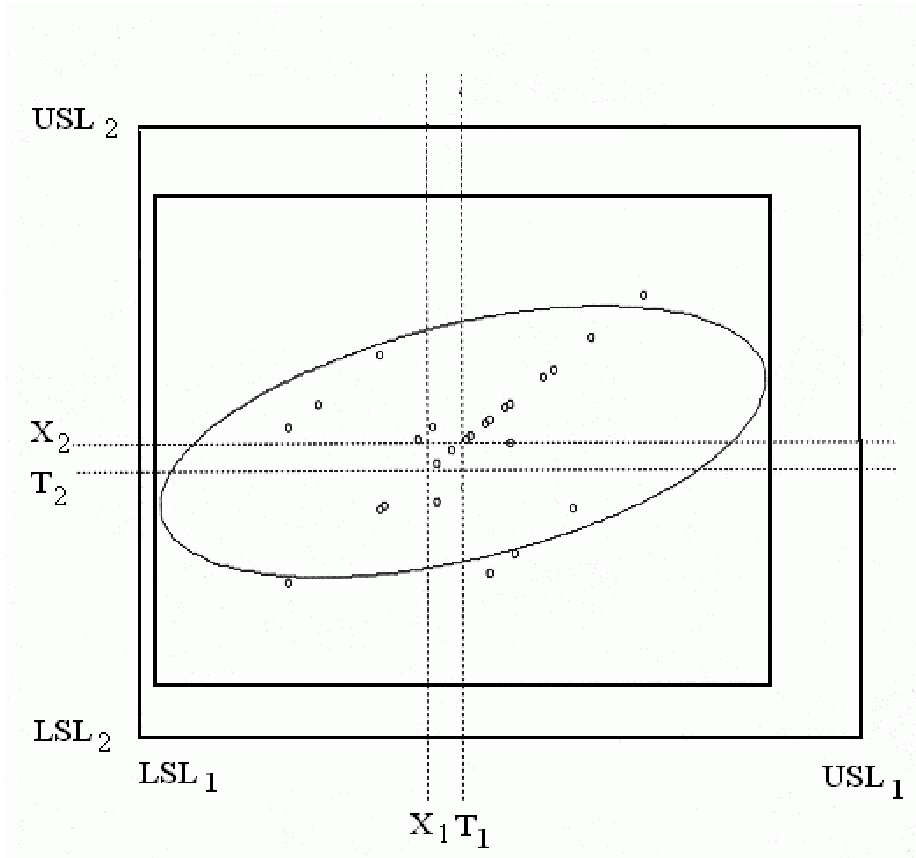


Fig. 2 View into the $x_1 0 x_2$ with the tolerance intervals for X_1 and X_2.

The variables x_1 and x_2 are means and define the point $(\bar{x}_1, \bar{x}_2)$ in the plane. T_1 and T_2 are target values for the quality characteristic X_1 and X_2 and define the ideal point (T_1,T_2). A test decides about the level of centralization. If the distance between the points $(\bar{x}_1, \bar{x}_2)$ and (T_1,T_2) is significant, the centralization is unsatisfactory.

10. 1 Is it possible to calculate a capability index at the job shop production?

When the capability is assessed at the jobshop production, the percentage use of the tolerance is calculated with the following formula

$$Qi = \frac{100(X_i - T)}{d}$$

where d is tolerance, Xi is the achieved value of the quality characteristic, T is the set (target) value of the quality characteristic and Q is the percentage use of the tolerance (here it is a symmetric tolerance).

Problem: The tolerance interval is described by LSL = 5, USL = 10 and T = 7.5. If X = 7 was achieved, then Q = -20 % use of the tolerance (the minus sign only says what the orientation of the achieved value x is, here it is to the left from T).
If X = 9.5, then Q = 80% and for X = USL=10 is Q = 100%. Qi is calculated for X_{max} and X_{min}.

The Capa software also calculates the average absolute value Qi, which is useful in case there is more than one measure taken. The Capa denotes the value in its result report as E/Q/. For two measures, X = 7 and X = 9.5, the average use of tolerance is E/Q/ = 50.

The capability index CpT is also calculated. The calculation is based on the original formula that is included in the Capa software. Here, for X = 7, is CpT = 5.0.

10.2 Is it possible to calculate a capability index when controlling calibration ?

Cpk index can be assigned to characteristics measured via pass / fail data (attribute type).

Problem: If the nonconformance rate is, for example, NC = 0.135 %, then it is possible to calculate Cpk = 1 (Capa, the chapter NoN distribution: Attributes).

Notes: 1) For Cpk < 0 it is printed: Cpk = 0.
2) $0 \leq NC \leq 100$