

# Calculating MMC Cpk

When there is a Maximum  
Material Condition or Least  
Material Condition True  
Position Specification

The following was developed in 1991. The then General Motors Fuel Handling Division had sent a request for quote with a drawing that had a MMC True Position feature that was designated with a “snow cone,” and the customer wanted a capability index. The problem was that each part has its own upper tolerance.

It has been shared with many since it was developed: Delphi, several 2nd and 3rd tier customers, recently Delphi and several Quality Managers and consultants through the Internet. This is the second revision, the first having been a spreadsheet example with some text included.

Marty Ambrose  
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Purpose:

To describe the method of calculating a capability index for a true position dimension that has a different tolerance for each part.

Scope:

Applies to those who must determine the capability index of a true position at Maximum Material Condition callout on a part drawing. Statistical methods - other than the Cpk calculations - and “normality” will not be discussed in this procedure. References to in-depth statistical methods can be found elsewhere in many available statistical reference books and literature. In addition, you must be familiar with Geometric Dimensioning and Tolerancing.

Definitions:

MMC - Maximum Material Condition - describes a part feature at its maximum virtual condition. Signified by  $\textcircled{M}$  on the drawing.

LMC - Least Material Condition - describes a part feature at its smallest virtual condition. It is fundamentally the reverse of MMC.

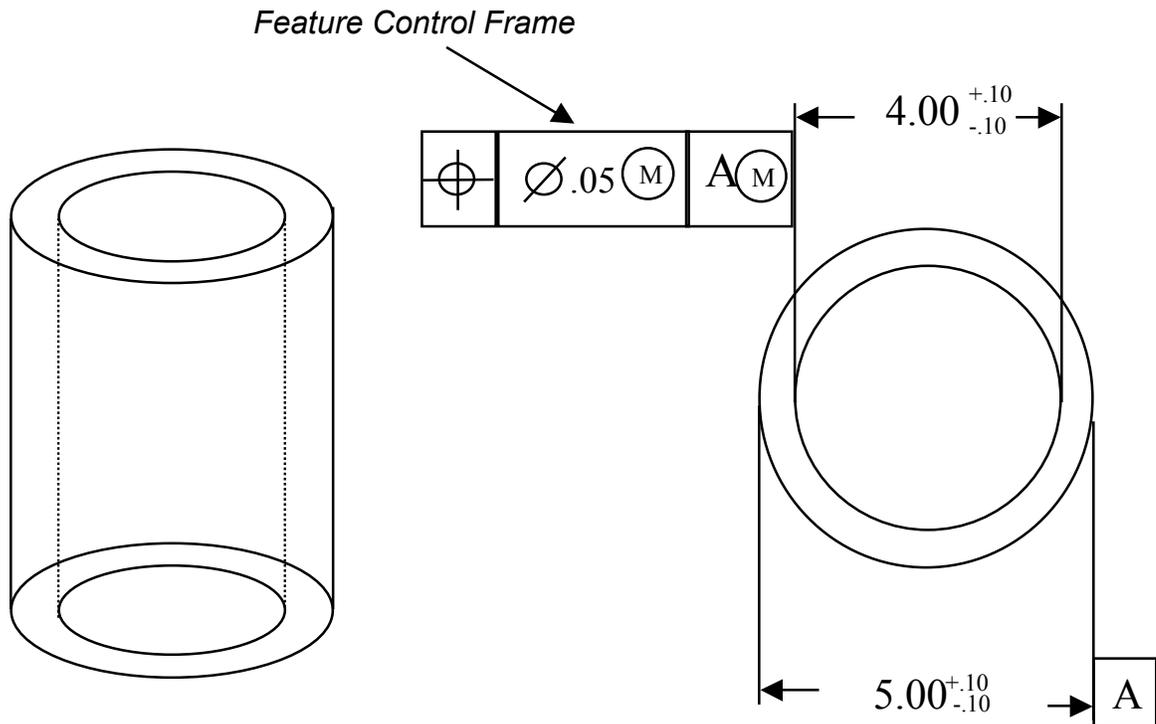
Signified by the GDT symbol  $\textcircled{L}$

*If one considers a donut, its maximum condition is when its hole is at its smallest, and its outside is at its biggest. Its least material condition is opposite.*

True Position - The intended location - and allowable variation of that location - of a feature of size. Signified by this GDT symbol  $\textcircled{\oplus}$

Bonus Tolerance - the amount of additional tolerance allowed when features detract from their maximum (or virtual) condition.

For this demonstration, use the part drawing example below. For the purpose of this procedure, we will assume the true position is “critical,” meaning you need to determine capability. To be brief, only 5 will be used.



In plain terms, in the drawing above, it states that the outside diameter (OD) is Datum A, and the inside diameter (ID) is allowed to have a positional variation or location of .05 with respect to Datum A (OD) when the ID is at its maximum condition and the OD is at its maximum condition. The OD is allowed to vary a total of .20 dimensionally, as is the ID.

The maximum condition of the part (using the donut approach,) is the ID is 3.90, the OD is 5.10, and the ID can be as far “off position” by as much as .05

Five parts are measured, the results listed:

<u>Part</u>	<u>OD</u>	<u>ID</u>	<u>ID Position</u>
1	5.023	4.022	0.140
2	5.055	4.086	0.105
3	5.045	3.955	0.025
4	4.955	4.110	0.021
5	5.090	4.050	0.068

At first glance, it appears there is a problem with part #1, #2, #4 and #5. In fact, part #4 is out of spec (ID), and normally would not be assessed. However, it is left in for explanation purposes. Parts #1, #2, #5 appear to also be out of position; let's see...

## Steps to Cpk Calculation

To calculate Cpk, several steps are needed. The first step is done, measuring the parts. The next step is to calculate the upper tolerance for each part. To do this, determine the bonus tolerance allowed for each part.

### 1. Calculate Bonus Tolerance

When calculating bonus, the dimension of each part feature referenced in the feature control frame is determined, then subtracted from the virtual (maximum) condition. Example - Part #1 above:

$5.10 - 5.023 = .077$  (the maximum condition minus the actual)

$4.022 - 3.90 = .122$ . (the actual minus the maximum condition)

<u>Part</u>	<u>OD</u>	<u>ID</u>	<u>Bonus</u>
1	$5.10 - 5.023 = .077$	$4.022 - 3.90 = .122$	$.077 + .122 = .199$
2	$5.10 - 5.055 = .045$	$4.086 - 3.90 = .186$	$.045 + .186 = .231$
3	$5.10 - 5.045 = .055$	$3.955 - 3.90 = .055$	$.055 + .055 = .110$
4	$5.10 - 4.955 = .145$	<b><math>4.110 - 3.90 = .200</math></b>	$.145 + .200 = .345$
5	$5.10 - 5.090 = .010$	$4.050 - 3.90 = .150$	$.010 + .150 = .160$

## 2. Calculate Each Part's Upper Tolerance

Next add in the true position actual tolerance as specified in the feature control frame. In this example, it is .05. If it was 0.00, there is no added calculation.

So, in the case of part #1,  $.199 + .05 = .249$ . This is the upper tolerance for #1.

<u>Part</u>	<u>Bonus</u>	<u>Position in FCB</u>	<u>Total Bonus (Upper Tolerance)</u>
1	0.199	.05	0.249
2	0.231	.05	0.281
3	0.110	.05	0.160
4	0.345	.05	0.395
5	0.160	.05	0.210

Notice that Part #4 should have had a bonus of 0.355. However, the ID is out of spec, and you cannot take any more bonus than the print allows. (Yes, this part is actually a reject, but left in for this explanation. The measured ID was 3.89, listed above, and the lower tolerance is 3.90.)

## 3. Calculate Percent of Tolerance Used

Next, compare the actual measured position of the ID to the allowed positional tolerance, that is, the total bonus allowed. Calculate how much of the calculated upper tolerance is actually used.

(Refer to the values on page 5)

Example: Part #1  $.140 / .249 = .5622$  or 56.22%

<u>Part</u>	<u>Actual ID Pos.</u>	<u>Bonus</u>	<u>Percent of Tolerance</u>
1	0.140	0.249	56.22%
2	0.105	0.281	37.37%
3	0.025	0.160	15.63%
4	0.021	0.395	5.32%
5	0.068	0.210	32.38%

#### 4. Cpk Calculation

Now that there is a method of determining the percent of tolerance used for each part, we then have a common upper tolerance limit - **100%**, **that is, 100% of the tolerance available. For the purposes of statistical software use, express the decimal equivalent of the percentage.**

For this example (including Part #4):

Average 29.38% (or .2938)

Std Dev .197591

Cpk is  $(UTOL - \bar{X}) / (3 * \text{Std DEV})$

Upper tolerance is 1.00 (or 100%)

$(1.00 - .2938) / (3 * .197591)$

$(.7062) / (.592774)$

Cpk - 1.19

## Appendix

The following is an example spreadsheet. It includes the data from above. The last column is the data that is to be analyzed. My experience with Statistical software is that using a spreadsheet to enter the appropriate data and then the formulas is easier than calculating manually and then entering the percent of tolerance. After entering all the data, I import the the column into a statistical software package.

For this example, the decimal is shown as a percent. When importing into a stats package, the values should be expressed in decimal format.

The Xbar and other calculations are also shown for demonstration. This would be accomplished by a statistical package. In the case of MMC or LMC, the upper tolerance is always 1 (1.00 or 100%)

act od	OD Bonus	act ID	ID Bonus	OD+ID Bonus	act id pos	OD+ID Bonus+.05	Act ID / Bonus=PCT of Tol
5.023	0.077	4.022	0.122	0.199	0.140	0.249	56.22%
5.055	0.045	4.086	0.186	0.231	0.105	0.281	37.37%
5.045	0.055	3.955	0.055	0.110	0.025	0.160	15.63%
4.955	0.145	4.110	0.200	0.345	0.021	0.395	5.32%
5.090	0.010	4.050	0.150	0.160	0.068	0.210	32.38%
						avg	29.38%
						std dev	0.19759134
						UTOL-avg	70.62%
						3*std dev	0.59277
						Cpk	1.19