

## Process Capability Studies

[The following is based in part on the paper "Process Capability Studies" prepared by M. Suozzi, Member of the Technical Staff, Hughes Aircraft Company, Tucson, Arizona, 27NOV90]

### Introduction

Process capability is an important concept for industrial managers to understand. The challenge in today's competitive markets is to be on the leading edge of producing high quality products at minimum costs. This cannot be done without a systematic approach and this approach is contained within what has been called "statistical quality control" or "industrial statistics." The segment of statistical quality control (SQC) discussed here is the process capability study.

So why is process capability so important? Because it allows one to quantify how well a process can produce acceptable product. As a result, a manager or engineer can prioritize needed process improvements and identify those processes that do not need immediate process improvements. Process capability studies indicate if a process is capable of producing virtually all conforming product. If the process is capable, then statistical process controls can be used to monitor the process and conventional acceptance efforts can be reduced or eliminated entirely. This not only yields great cost savings in eliminating non-value added inspections but also eliminates scrap, rework and increases customer satisfaction. The benefits of performing process capability studies are certainly worth the effort in the long run.

After a process capability study has been performed, a process will be classified as either capable or incapable. When the process is not capable of producing virtually all conforming product, the process is said to be incapable and acceptance sampling procedures (or 100% inspection) must remain part of the process. Figure 1 shows the process output when compared to the specification limits for a typical incapable process.

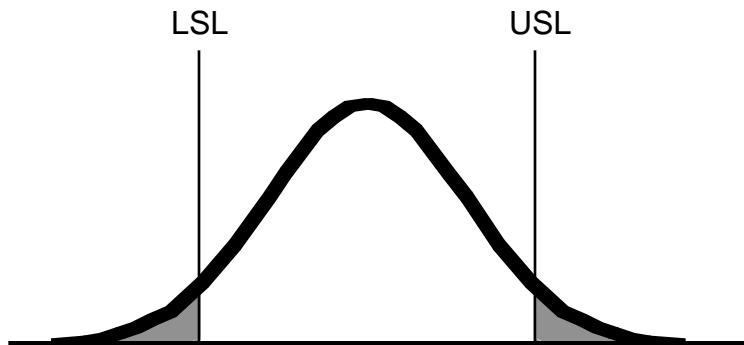


Figure 1

Processes may also start out as capable but changes over time to have more variability. In addition, the process mean may also shift placing the process too close to one of the specification limits. Both increases in process variability and shifting of the mean may result in once capable processes becoming incapable.

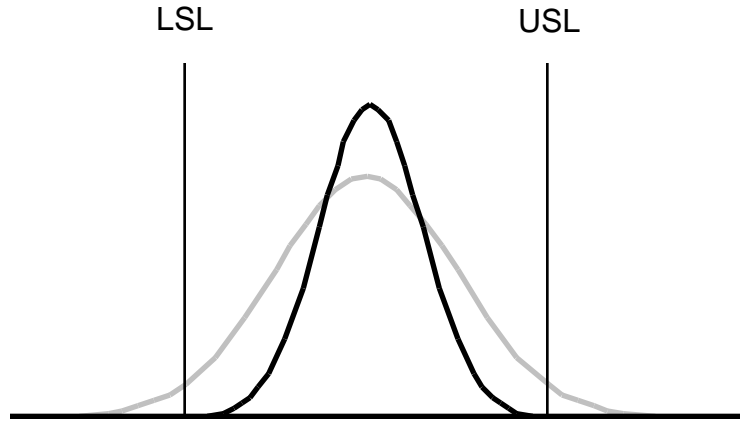


Figure 2

Figure 2 illustrates a once capable process that, over time, has become incapable due to increased variation. Figure 3 illustrates the effects of process capability due to shifting in the mean with variation remaining the same.

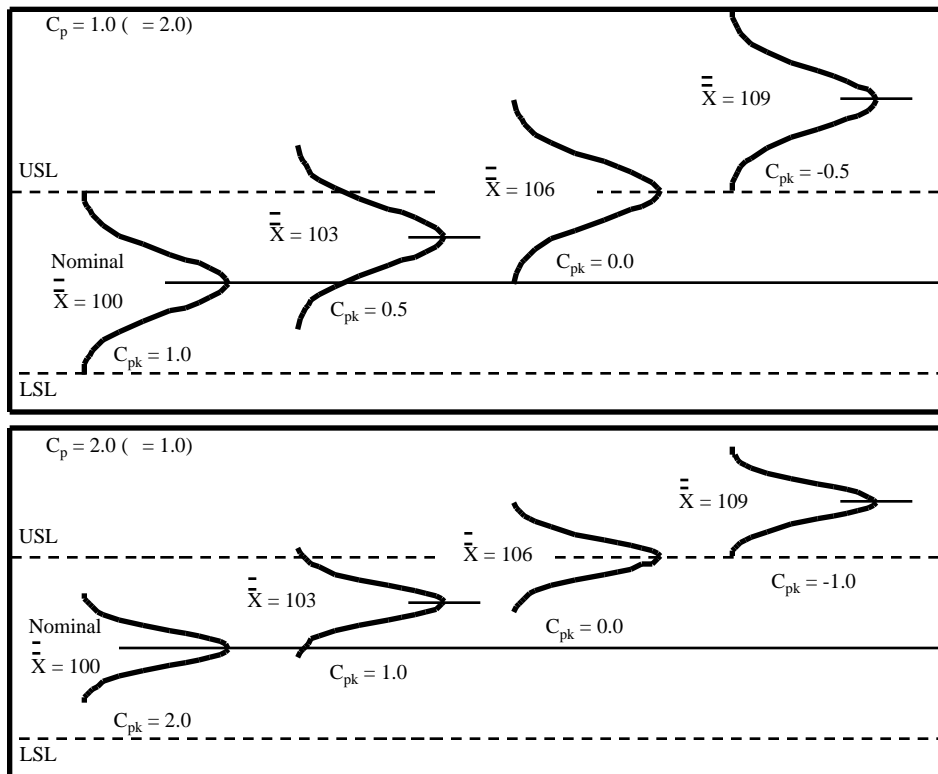


Figure 3

## Definitions

Process: Process refers to any system of causes; any combination of conditions which work together to produce a given result.

Process Capability: Process capability refers to the normal behavior of a process when operating in a state of statistical control. It refers to the inherent ability of a process to produce similar parts for a sustained period of time under a given set of conditions.

Process Capability Indices: Process capability can be expressed as percent nonconforming or in terms of the natural spread related to the specification spread.

Process Capability Study: A process capability study is a systematic procedure for determining the capability of a process. The process capability study may include studies to improve the process and in turn the capability of the process. Process capability studies are usually performed as part of a process certification effort or a process optimization effort.

## Process Capability Study Steps

The general process capability study steps are:

### 1. Select Critical Parameters

Critical parameters need to be selected before the study begins. Critical parameters may be established from drawings, contracts, inspection instructions, work instructions, etc. Critical parameters are usually correlated to product fit and/or function.

### 2. Collect Data

A data collection system needs to be established to assure that the appropriate data is collected. It is preferable to collect at least 60 data values for each critical parameter. If this is not possible, corrections can be made to adjust for the error that is introduced when less than 60 data values are collected. Significant digits for each data should be the number of significant digits required per the specification limits plus one extra significant digit to assure that process stability can be evaluated.

### 3. Establish Control over the Process

A distinction between product and process should be made at this point. The product is the end result from the process. The product may be a physical item (Example: fabricated part) or a service (example: typing a report). One may control the process by measuring and controlling parameters of the product directly or measuring and controlling the inputs to the process (once correlation between the process inputs and product critical parameters have been established). It is ultimately desirable to establish control over the process by controlling the process inputs. On the other hand, process capability indices are always performed using the critical parameters of the product.

Calculation of predictable process capability indices is dependent on the statistical control of the process. If the process is not in statistical control, then the results of the study are subject to fluctuate unpredictably. The statistical control of the process can be studied using control charts (usually Xbar-R charts).

#### 4. Analyze Process Data

To calculate the process capability indices, estimates of the process average and dispersion (standard deviation) must be obtained from the process data. In addition, the formulas for process capability indices assume that the process data came from a normal statistical distribution. It is important that one prove that the data is normally distributed prior to reporting the process capability indices because errors in misjudgment can lead to the same undesirable effects as listed in Step 1. Methods for handling non-normal data and formulas for several process capability indices will be addressed in separate sections.

#### 5. Analyze Sources of Variation

Study of the component sources of variation and their magnitudes may range from simple statistical tests to complex experimental designs carried out over a long period of time. If possible, tests should be kept simple. Analyzing sources of variation involves determining what process factors affect the natural process spread (process variation) and the process centering. With this knowledge, it may be possible to improve the process' capability. Analyzing sources of variation always involve careful planning and data collection.

#### 6. Establish Process Monitoring System

Once the process capability indices indicate a capable process, a routine process control technique should be employed to assure that the process remains stable. This may be done by a variety of methods such as establishing a statistical process (SPC) program.

The process capability indices should also be periodically recalculated to assure the process mean and spread has not significantly changed.

### **Process Capability Indices**

#### Calculating the Process Mean (Xbar)

$$Mean = Xbar = \frac{\sum Xi}{n}$$

Where n is the sample size and Xi is the data value.

#### Calculate the Process Spread (Standard Deviation, s)

$$s = \sqrt{\frac{\sum (Xi - Mean)^2}{(n - 1)}}$$

To obtain an accurate estimate of the process spread (standard deviation); at least 60 data points are needed. If less than 60 data points are available, use the following formula with the error correction factors given in Table 1.

$$S_{corrected} = \frac{S}{C_4}$$

### **Standard Deviation Correction Factors**

n	C <sub>4</sub>
15	0.9823
16	0.9835
17	0.9845
18	0.9854
19	0.9862
20	0.9869
21	0.9876
22	0.9882
23	0.9887
24	0.9892
25	0.9896
30	0.9914
35	0.9927
40	0.9936
45	0.9943
50	0.9949
55	0.9954

Table 1

### **Calculating Process Capability Indices**

There are many process capability indices available. Presented here are several of the most common indices.

#### 1. C<sub>p</sub>

This is a process capability index that indicates the process' potential performance by relating the natural process spread to the specification (tolerance) spread. It is often used during the product design phase and pilot production phase.

$$C_p = \frac{\text{SpecificationRange}}{6s} = \frac{(USL - LSL)}{6s}$$

Where USL is the Upper Specification Limit and LSL is the Lower Specification Limit.

## 2. $C_{pk}$ (2-Sided Specification Limits)

This is a process capability index that indicates the process actual performance by accounting for a shift in the mean of the process toward either the upper or lower specification limit. It is often used during the pilot production phase and during routine production phase.

$$C_{pk} = \text{Minimum} \frac{USL - Mean}{3s} ; \frac{Mean - LSL}{3s}$$

$$C_{pku} = C_{pk} \text{ (Upper Specification Limit)}$$

$$C_{pkl} = C_{pk} \text{ (Lower Specification Limit)}$$

## 3. $C_{pk}$ (1-Sided Specification Limits)

$C_{pk}$  can be calculated even if only one specification limit exists or if a minimum/maximum is specified.

- a.  $C_{pk(max)}$ :  $C_{pk}$  for Upper Specification Limit or maximum specified.

$$C_{pk(max)} = \frac{(USL - Mean)}{3s}$$

- b.  $C_{pk(min)}$ :  $C_{pk}$  for Lower Specification Limit or minimum specified.

$$C_{pk(min)} = \frac{(Mean - LSL)}{3s}$$

## 4. Capability Index for Attributes Data

Ford Motor Company established a capability index for attributes (go/no-go) data.

- a. No failures to meet specifications

$$Capability\% = 100(0.5)^{\frac{1}{n-1}}$$

- b. With failures to meet specifications

$$Capability\% = 100 \left( 1 - \frac{F + 0.7}{n} \right)$$

Where F is the number of failures.

Table 2 gives the equivalent  $C_p$  value corresponding to Capability %.

Equivalent $C_p$	Capability %
0.50	86.64
0.62	93.50
0.68	96.00
0.75	97.50
0.81	98.50
0.86	99.00
0.91	99.35
1.00	99.73
1.33	99.994

Table 2

5. Capability Index for use with Target Values,  $C_{pm}$

$C_{pm}$  is used when a target value other than the center of the specification spread has been designated as desirable.

$$C_{pm} = \frac{C_p}{1 + \frac{(\bar{X} - T)^2}{s^2}}$$

Where T is the process target value other than the center of the specification.

6. Capability index for “Smaller is Better” quality characteristics,  $C_r$ .

$C_r$  expresses  $C_p$  in a ratio format and is often used as a “smaller is better” index.

$$C_r = \frac{100}{C_p}$$

The most common process capability indices used are  $C_p$  and  $C_{pk}$ . Other indices are used for special situations, as indicated.

### Process Capability Analysis from Control Charts

Control charts are used to monitor the process after the initial process capability evaluation. However, periodic re-evaluation of process capability is recommended to assure that the process mean has not shifted and that the process variation has not increased. The control chart may be used to obtain an estimate of the standard deviation and the process mean for use in the process capability formulas. Use the following formulas to obtain the mean and standard deviation estimates with the appropriate error correction factors from Table 3.

$$\begin{aligned}\text{Mean} &= \bar{X} \\ &= \frac{\bar{R}}{d_2} \\ &= \frac{\bar{s}}{c_4}\end{aligned}$$

In this case, the grand mean ( $\bar{X}$ ) is used as an estimate for the process mean ( $\bar{X}$ ).

### Standard Deviation Correction Factors

n Subgroup Sample Size	$d_2$	$c_4$
2	1.128	0.7979
3	1.693	0.8862
4	2.059	0.9213
5	2.326	0.9400
6	2.534	0.9515
7	2.704	0.9594
8	2.847	0.9650
9	2.970	0.9693
10	3.078	0.9727
11	3.173	0.9754
12	3.258	0.9776
13	3.336	0.9794
14	3.407	0.9810
15	3.472	0.9823

Table 3

### Process Capability Related to Percent Nonconforming

Process capability indices indicate the inherent capability of a process but what does this mean with respect to yield and percent nonconforming that may be expected from the process?

Many process capability indices ( $C_p$ ,  $C_{pk}$ ) may be expressed in terms of percent nonconforming by using the standard normal distribution tables. Presented here is the procedure to convert  $C_p$  and  $C_{pk}$  into a percent nonconforming value. The procedures presented here assume that the normal tables used define the area less than or equal to the Z-Score. Other tables need to be interpreted appropriately.



### Converting $C_p$ to Percent Nonconforming

Step 1. Calculate the Z-Score

$$Z\text{-Score} = 3C_p$$

The Z-Score is the Z percentage point from the standard normal distribution tables. In fact,  $C_p$ , and  $C_{pk}$  assumes that the process data follows a normal distribution.

Step 2. Use the Z-Score to find the Z-Score curve area value in the standard normal table.

Since the Z-Score is the Z percentage point, the area under the normal distribution curve can be looked up directly in the normal tables.

Step 3. Convert the Z-Score curve area to the process percent nonconforming and process yield.

$$\text{Process Percent Nonconforming} = (1.0 - Z\text{-Score Curve Area})(2)(100)$$

The process percent nonconforming is the long term percent nonconforming that can be expected from the process if it is allowed to operate at the current capability.

The long term process yield is:

$$\text{Process Yield} = 100 - \text{Process Percent Nonconforming}$$

The process yield indicates the long term process yield that can be expected from the process if it is allowed to operate at the current capability.

### Converting Percent Nonconforming to $C_p$

Step 1. Calculate:

$$1.0 - \frac{\text{Percent Nonconforming}}{(100)(2)}$$

This is the Z-Score curve area that will be used to obtain the Z-Score percentage point from the normal tables.

Step 2. Using the standard normal tables as the quantity obtained in Step 1, obtain the Z-Score

Step 3. Convert the Z-Score to the equivalent  $C_p$

$$C_p = \frac{Z - \text{Score}}{3}$$

To obtain the percent nonconforming from  $C_{pk}$ , the percentages from both tails ( $C_{pku}$  and  $C_{pkl}$ ) needs to be calculated and the results added.

### Converting $C_{pku}$ and $C_{pkl}$ to Percent Nonconforming

Step 1. Calculate the Z-Score

$$Z\text{-Score} = 3C_{pk}$$

The Z-Score is the Z percentage point from the standard normal distribution tables. In fact,  $C_p$  and  $C_{pk}$  assume that the process data follows a normal distribution.

Step 2. Use the Z-Score to find the Z-Score curve area value in the standard normal table.

Since the Z-Score is the Z percentage point, the area under the normal distribution curve can be looked up directly in the normal tables.

Step 3. Convert the Z-Score curve area to the process percent nonconforming and process yield.

Process Percent Nonconforming for One Tailed ( $C_{pku}$  or  $C_{pkl}$ ):

$$(1.0 - Z\text{-Score Curve Area})(100)$$

Step 4: Repeat Steps 1-3 for the other tail ( $C_{pku}$  or  $C_{pkl}$ ).

Step 5: Sum to obtain the total percent nonconforming

### Converting Percent Nonconforming to $C_{pk}$

Step 1. Calculate:

$$1.0 - \frac{\text{Percent Nonconforming}}{(100)}$$

This is the Z-Score curve area that will be used to obtain the Z-Score percentage point from the normal tables.

Step 2. Using the standard normal tables as the quantity obtained in Step 1, obtain the Z-Score

Step 3. Convert the Z-Score to the equivalent  $C_{pk}$

$$C_{pk} = \frac{Z\text{-Score}}{3}$$

**The long term process percent nonconforming (and long term process yield) is the bottom line result sought in a process capability study.**

Process yield information can be converted into dollars, the ultimate management tool for assessing a company's competitive position. With good process capability indices, the product is being produced with virtually zero nonconforming.

Table 4 shows several  $C_p$  values and their equivalent percent nonconforming and parts per million (PPM) values.

**$C_p$  Related to Percent Nonconforming and Parts per Million (PPM)**

$C_p$	Percent Nonconforming	Parts per Million (PPM)
0.50	13.36	133,600
0.62	6.50	65,000
0.68	4.00	40,000
0.75	2.50	25,000
0.81	1.50	15,000
0.86	1.00	10,000
0.91	0.65	6500
1.00	0.27	2700
1.33	0.006	60

Table 4

Table 5 shows several  $C_{pk}$  values and their equivalent percent nonconforming and parts per million (PPM) values.

**$C_{pk}$  Related to Percent Nonconforming and Parts per Million (PPM)**

$C_{pku}$ or $C_{pkl}^*$	Percent Nonconforming	Parts per Million (PPM)
0.50	6.68	66,800
0.51	6.50	65,000
0.58	4.00	40,000
0.65	2.50	25,000
0.72	1.50	15,000
0.78	1.00	10,000
0.83	0.65	6500
1.00	0.135	1350
1.33	0.003	30

Table 5

\*To obtain the total percent nonconforming, sum the percent nonconforming values for  $C_{pku}$  and  $C_{pkl}$ . To obtain PPM, multiply percent nonconforming by 10,000.

## Process Capability Index Standards

The current process capability index standards for  $C_p$  and  $C_{pk}$  are:

$$C_p > 1.33$$

$$C_{pk} > 1.33$$

This value is selected to be 1.33 for a number of reasons. Processes are seldom truly static and this value allows for small process shifts. For example, if  $C_{pk}$  changes from a value of 1.0 to 0.67 (a shift a one standard deviation), the process experiences an increase from 0.27% to 4.55% nonconforming. In addition, serial processes usually contribute an additive effect for nonconformances and high process capability indices assure an acceptable end-of-line  $C_{pk}$ .

## Process Capability Reevaluation

Process capability should be reevaluated periodically to assure that the process mean has not shifted and that the process variation has not increased. The recommended minimum frequency for reevaluation is six months.

## Process Capability Study Assumptions

There are two assumptions that must be met prior to performing a process capability study:

1. The performance measure data reflects statistical control when plotted on a control chart (usually Xbar-R chart)

This is necessary because if the process is not in a state of statistical control, then the results of the process capability study is transient in nature and decisions concerning the process should not be made based on the outcome of the study.

2. The performance measure data is distributed normally.

The section on process capability indices and estimating process fraction nonconforming discussed obtaining an estimate of the process fraction nonconforming using the standard normal tables. If the data were not distributed normally, one may be underestimating or overestimating the true process fraction nonconforming. There is no danger in overestimating the process fraction but there is danger when the process fraction is underestimated. The risk lies in the fact that acceptance sampling may be reduced when the study indicates a capable process. If the process is truly not capable, an unacceptable amount of nonconforming product may reach the factory floor and costs may increase due to scrap and rework. This could be devastating for a fixed price contract that was bid based on low nonconformance rates.

## **Normality Testing**

There are several analysis methods that can be used to determine if the data is normally distributed or not. These include chi-square goodness-of-fit, K-S Test and normal probability plots. Histograms may also be used, but plotting the data on a histogram and eyeballing the fit may not prove accurate.

## **Methods for Handling Non-Normal Data**

If the data is non-normal, another method must be used in order to have a valid process capability study.

1. Transformation of the data and calculate the process capability indices using the transformed data.

This requires knowledge of the appropriate transformations. It is also time consuming because the normality assumptions must be checked after each transformation.

2. Utilize Johnson or Pearson distribution fitting techniques and determine the process capability from the appropriate percentage points of the distribution.
3. Ford Motor Company demonstrates what they call Estimated Accumulative Frequency and the Mirroring Technique to handle non-normal distributions.

## **Process Capability and Statistical Process Control**

Process capability and statistical process control (SPC) are completely independent. Both process capability and process statistical control are necessary in order to certify a process and reduce or eliminate conventional inspection techniques.

## **Evaluation of Capability**

Figure 4 reflects a process in control; that is, the sample average values are within the control limits. The curve at the left of the chart represents the distribution of the sample averages (the control limits are the "tails" of this distribution, specifically at plus and minus three standard deviation units of the sample averages).

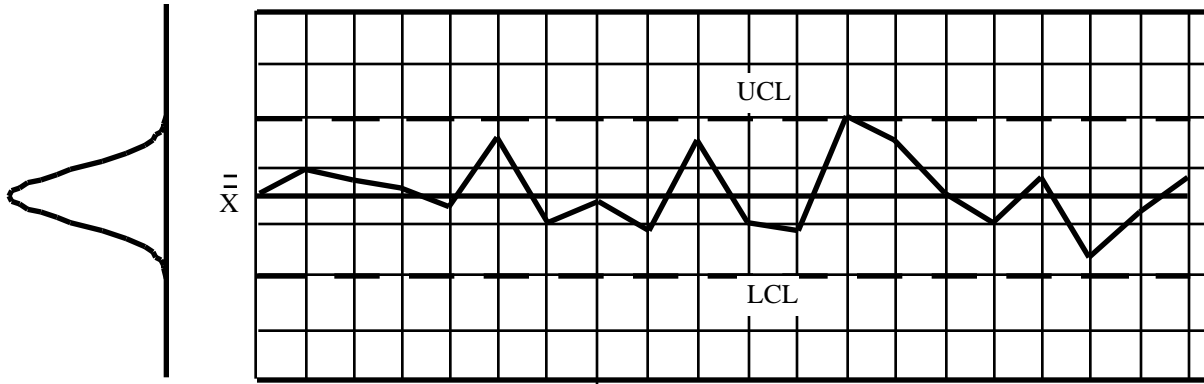


Figure 4

Figure 5 represents the distribution of the entire population of parts. Note that the distribution of the population is wider than the distribution of the sample averages, by a factor equal to the square root of the sample size. The “tails” of the distribution of the entire population reflect the limits of the process variation. The points at plus and minus three standard deviation units are often called the “natural process limits.”

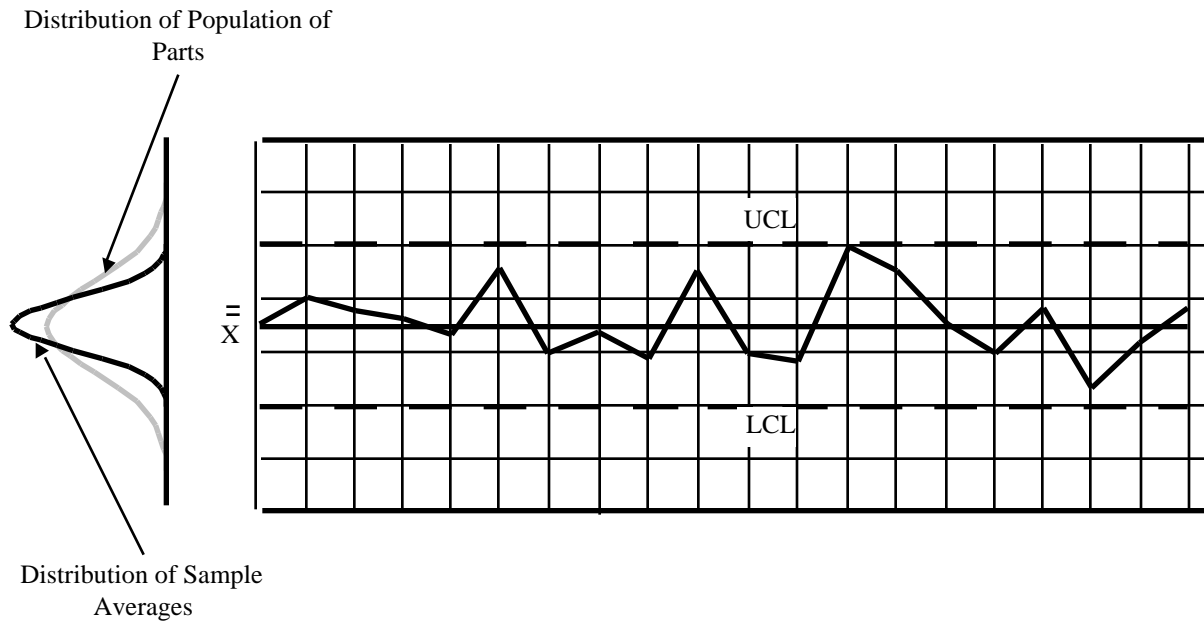


Figure 5

Figure 6 illustrates an example comparing the natural process limits to the product specification. The specification limits are slightly wider than the control limits. Even though all of the sample averages are within the control limits, there will still be a substantial proportion of parts in the entire population that will not meet specification (indicated by the shaded area under the curve). This process can be said to be in control but not capable.

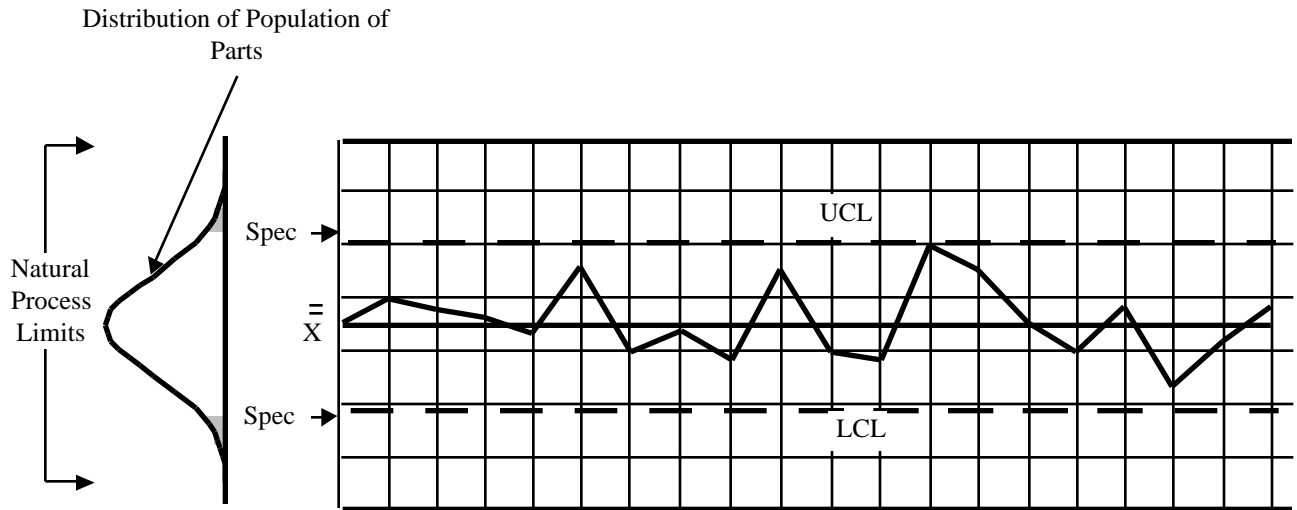


Figure 6

Figure 7 illustrates the situation when the natural process limits are within the specifications. This process can be judged as both in control and capable.

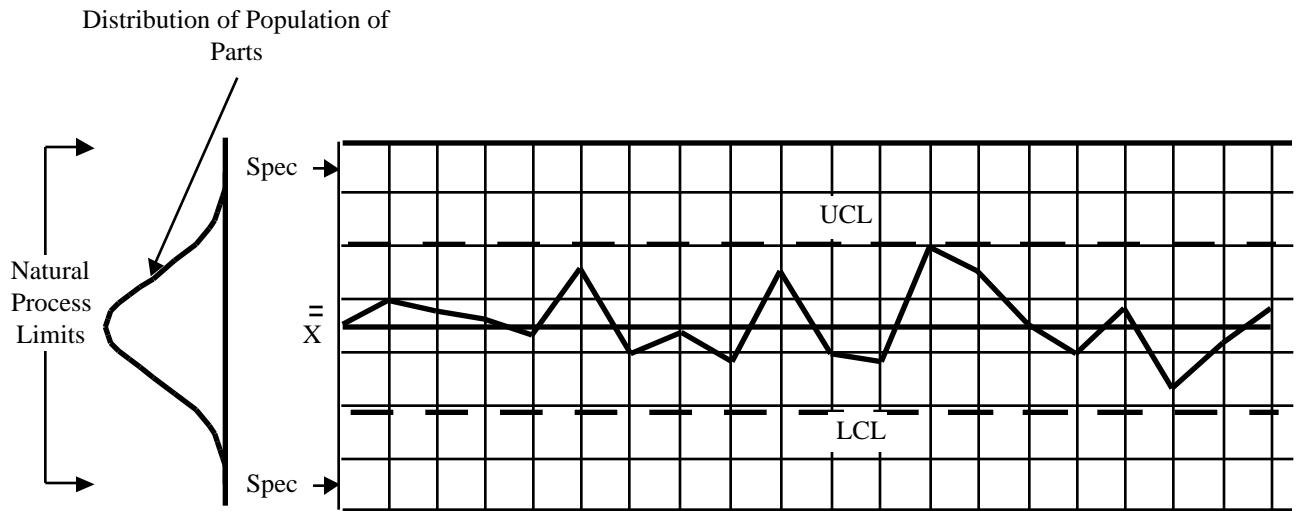


Figure 7

## End Stuff

The derivation presented below from Grant, Eugene L., and Leavenworth, Robert S., Statistical Quality Control is presented to explain the use of the sigma correction factors ( $c_4$ ) in Table 1.

An unbiased estimator of  $\sigma$  (sigma or population standard deviation) obtained from  $s$  (sample standard deviation) is given as:

$$= \frac{s - \bar{bar}}{c_4}$$

$$_{rms} = S * \sqrt{\frac{n - 1}{n}}$$

$$_{hat} = \frac{_{rms}}{c_4 * \sqrt{\frac{n - 1}{n}}}$$

$$_{hat} = \frac{S * \sqrt{\frac{n - 1}{n}}}{c_4 * \sqrt{\frac{n - 1}{n}}}$$

$$_{hat} = \frac{S}{c_4}$$