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2.3 More terminology

This section provides, for reference, specific definitions of some of the other terms used in error analysis. It is included here because this terminology is sometimes in conflict with nonscientific usage and is not always used consistently even in scientific papers.

With many of these terms, it is necessary to distinguish between the characteristics of a ***parent*** distribution and the estimates of those characteristics obtained from a specific sample from the parent population. For example, one may want to estimate characteristics of the parent population from measurements taken on only a specific subset from that population. A common convention, followed here, is to use Greek letters for population characteristics and Roman letters for sample characteristics. Thus, for example, $\overline{x}$ will denote the average of a set of measurements, but μ will denote an average characteristic of the underlying population.

Precision is a measure of reproducibility or scatter in the results, without regard for the accuracy of the result. It is a measure of random error only; systematic errors will not affect the precision of a result, although they do affect the accuracy.

The ***mean*** of a set of measurements $\{x_1, x_2, \ldots x_N\}$ is the average:

$$\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i. \qquad (2.1)$$

The ***expectation value*** of a quantity is the value expected if averaged over the entire parent population, and will be denoted by angle brackets: $\langle\rangle$. For example, the mean in the parent population that corresponds to the sample mean x is

$$\mu = \langle x \rangle = \lim_{N\to\infty}(\overline{x}). \qquad (2.2)$$

There is an important distinction to be made between the standard deviation characterizing the random error of a measurement and the standard deviation characterizing a set of accurate observations and hence reflecting physical reality. The latter is often encountered in experimental research, and pertains to the natural variability in the parameter being measured. The former represents the precision with which a constant value of that parameter could be measured in a particular experiment. For example, in experiments using airborne instrumentation variance spectra for measured variables seldom show evidence of noise except at low levels that correspond to digitization noise. This indicates that random measurement errors seldom contribute significantly to the uncertainty in such a measurement. However, there usually is high natural variability that causes repeated sets of measurements in presumably identical conditions to vary significantly, and the standard deviations among repeated measurements of, for example, fluxes of water vapor are large. This standard deviation reflects natural variability, not the random error in the measurement. It results from the variability of particular samples about the underlying population mean, and that variability would still characterize measurements from error-free sensors.

The ***median*** is the value that divides the population into equal halves; i.e., half the members lie above and half below the median. The ***most probable value*** is that observed most frequently, sometimes referred to as the ***mode*** of a distribution. As an example, the expected distribution of time intervals between randomly occurring events is

$$N(t) = N_0 e^{-t/\tau} \qquad (2.3)$$

where $N(t)$ is the number of events per time interval that occur at time t, N_0 is the total number of events, t is the time, and τ is a time constant characterizing the process. For this distribution, the mean time is τ, the median time is $\tau \ln(2)$, and the mode occurs for t=0.

A ***deviation*** δ is the difference between a specific measurement or value and the mean. The ***standard deviation*** σ is the "root-mean-square" value of the deviations, obtained from

$$\sigma = \left[\lim_{N\to\infty}\left(\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2\right)\right]^{\frac{1}{2}}. \qquad (2.4)$$

For a sample of measurements, the conventional estimate s of the population standard deviation σ is

$$s = \left[\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\overline{x})^2\right]^{\frac{1}{2}}. \qquad (2.5)$$

The ***variance*** is the average of the squares of the deviations, or the square of the standard deviation.

Exercise 2.1: Show that the root-mean-square value of the deviations is less when calculated relative to the sample mean than it would be when calculated relative to any other quantity. This property shows that the mean of a sample of measurements provides a better measure, in a least-squares sense, of that value than any other quantity that could be calculated from those measurements.

Exercise 2.2: Show that an alternate formula for the sample standard deviation is

$$s = \left[\frac{1}{N-1}\sum_{i=1}^{N}(x_i^2 - \overline{x}^2)\right]^{\frac{1}{2}}. \quad (2.6)$$

This form has the computational advantage that all quantities can be calculated in one pass through the data, while the preceding form requires two passes, one to calculate the mean and the second to calculate the deviations from that mean.

If x_j is a possible observation, the observed fraction of observations having the value x_j is $P(x_j)=N(x_j)/N$ where N is the total number of observations and $N(x_j)$ is the number having value x_j. The underlying population distribution function is then

$$\Phi(x_j) = \lim_{N\to\infty} P(x_j). \quad (2.7)$$

The preceding quantities can then be expressed in terms of the distribution function; for example, the mean is

$$\mu = \sum_{j=1}^{N} x_j \Phi(x_j) \quad (2.8)$$

and the variance is

$$\sigma^2 = \sum_{j=1}^{N}(x_j - \mu)^2\Phi(x_j) = \left(\sum_{j=1}^{N} x_j^2\Phi(x_j)\right) - \mu^2 = \langle x^2\rangle - \mu^2. \quad (2.9)$$

The extensions to continuous distribution functions are these:

$$\sum_j P(x_j) \to \int P(x)dx \quad (2.10)$$

$$\mu = \int_{-\infty}^{\infty} x\Phi(x)\,dx \quad (2.11)$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\Phi(x)dx. \quad (2.12)$$

Similarly, the expectation value for any function f of measurable characteristics x is

$$\langle f(x)\rangle = \int f(x)\Phi(x)\,dx \qquad (2.13)$$

where ***x*** can be a set of variables and the multidimensional integration must then cover all possible values of ***x*** .

Other characteristics sometimes cited are the ***probable error*** , the magnitude of the deviation exceeded by 50% of the deviations, and the ***average deviation,*** the expectation value for the absolute magnitude of the deviations. For a Gaussian distribution, the probable error, average deviation, and standard deviation have the ratios 0.674:0.800:1.

If the distribution in measurement errors follows a known probability distribution, then ***confidence intervals*** determined from that distribution can be used to obtain quantitative estimates of probabilities associated with such errors. It is this relationship that establishes the often used correspondence between standard deviation and probability, for the Gaussian distribution. Specifically, measurements falling more than two standard deviations ($\pm 2\sigma$) from the true value are expected with about 0.05 probability, so 2σ limits correspond to approximate limits providing 95% coverage. Other distribution functions can be treated in the same way.

Exercise 2.3: Show that conversion from analog to digital representation of data introduces a random error component of $\delta^2/12$ to the variance in a measurement, where δ is the digital resolution.

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