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## METROLOGY

## Know Your Uncertainty

Understanding and documenting measurement uncertainty is key to gage calibration.

By Henrik S. Nielsen, Ph.D.
The process of developing uncertainty budgets requires a manufacturer to first find the causes of measurement uncertainty and quantify them. The two questions a manufacturer must ask, when identifying uncertainty contributors are, "How do I know what to look for?" and "What happens if I get it wrong?"

The first question is the most critical one. A manufacturer must understand the measuring process being evaluated, and a certain amount of experience, to identify and quantify uncertainty contributors. There are a couple of techniques that can help in systematically searching for uncertainty contributors.


When collecting uncertainty contributors, it is important to know the areas where they may be lurking. Thes e10 areas are th most ocmmon a manufacturer should use in looking for uncertainty contributors.

## Around in circles

The "circle of contributors" technique involves using a list of areas where uncertainty contributors may be lurking as an aid in searching for them. The list is usually depicted as a circle. The listed items are not the uncertainty contributors,

## TECH TIPS

* Use a formal technique to systematically search for uncertainty contributors.
* The operator model looks at the measurement process as a series of operations. Two models are conceptual, one is actual.
* Putting numbers to uncertainty contributors, to quantify them, can be done using statistical tools only or using other information such as calibration certifications, manufacturers' specifications, etc.
* Begin evaluations with the assumption that experimental data, to use statistical tools, is not available.
rather they are the areas in which the uncertainty contributors can be found. These areas can include:
- environment
- the reference element of the measuring equipment
- the measuring equipment
- measurement setup
- software and calculations
- the metrologist
- measuring object
- definition of the measurand
- measuring procedure
- physical constants.

The way the circle of contributors is used is that as an uncertainty budget is developed, the manufacturer asks, of each of the areas, "Are there any contributors here that apply to my measurement process?" Once the contributors have been identified, it's important to quantify how much each adds to the budget. But before doing that, it's important for the manufacturer to consider the consequences of what happens if he gets the identification of the uncertainty contributors wrong.

If the uncertainty contributors are incorrectly identified, one will either overestimate or underestimate the uncertainty. While this is not ideal, it is much better than the alternative, which is being totally in the dark about the uncertainty of the measurements.

Experience shows that with just a little bit of practice most people who have a basic knowledge of metrology can successfully identify the three to five largest uncertainty contributors. That is all it takes to make an uncertainty budget that is within $20 \%$ of the correct value, which is close enough to make it a valuable tool.

## Using the operator

Using a formal technique to systematically search for uncertainty contributors will help a manufacturer to avoid overlooking any possible contributor to uncertainty.

The "operator model" is a relatively recent concept developed by ISO Technical Committee 213. In short, the idea is to model the measurement process as a series of operations. For example, tracing a surface with a mechanical probe tip is an operation that transforms the information in the surface itself into the information the probe tip can "see." If a different probe tip radii is used, the operation is changed and different results occur, because a larger radius can "see" fewer details in the surface than a smaller radius.

These operations are similar to mathematical operations such as addition, subtraction, multiplication, and division. They present the same challenge as when two different operators give the same result for the same input using different methods, such as $2+2=4$ and $2 \times 2=4$. There may be difficulty in understanding and quantifying the differences because the result is the same, but the operations are different. For example, if there is a surface without much fine structure, such as the patches usually supplied with surface finish instruments to
help with adjustments, a change in tip radius from 2 to 10 microns does not result in much change in the measured roughness. For other surfaces with more fine structure, such as ground surfaces, it can make a big difference.


The operator model looks at the measurement process as a series of operations. A typical manufacturer would begin evaluating his measureent process by comparing it to the conceptual, perfect, then compare that conceptual, perfect to the conceptual, intended. Finally, the manufacturer would compare the conceptual, intended to the actual operator, which takes into account the real world.

In the operator model, there are three different operators, and it is the difference between these operators that causes the uncertainty. The definition uncertainty is the lowest uncertainty any measurand can be measured with, and the total uncertainty is given by the difference between the actual operator used in a measurement and the conceptual, perfect operator defining the measurand.

The "conceptual, perfect operator" is the definition of the measurand or what is supposed to be measured. The term is awkward, but "conceptual" means that this is a theoretical operator, and "perfect" means that it defines the true value.

Standards define the conceptual, perfect operator for what is supposed to be measured. Sometimes, the standards committee has not done a very good job of defining the measurand. If, in the previous example, the standard did not say anything about what tip radius to use, then all tip radii are in accordance with the conceptual, perfect operator. But, because the different tip radii will give different roughness values, which are all true according to the definition, there is a definition of uncertainty.

The definition of uncertainty comes from ambiguities in the definition of the measurand. This is probably the most overlooked uncertainty contributor.

The "conceptual, intended operator" is a theoretical operator, but it is one intended for use. It is the one built into the measuring equipment and measurement process, if everything went as planned and all parts of the equipment were physically perfect.

If it is assumed that the definition of the measurand says that surface finish is defined by the profile obtained when using a 2-micron tip radius, and the instrument had a nominal tip radius of 10 microns, then the difference in result comes from the nominal difference in tip radius and in operator, resulting in intentional uncertainty.

While few manufacturers will agree that they make measurements that are intentionally uncertain, that is, in effect, what they do when they deviate from the definition of what is to be measured.

Intentional uncertainty is not necessarily bad. Accepting a certain level of uncertainty may allow for more cost effective measurements. If it is known up front what the added uncertainty is, and it is acceptable, intentional uncertainty can be used to manage uncertainty and measurement cost.

## The actual operator

The "actual operator" is used in the actual measurement. It is the real-world implementation of the conceptual, intended operator. The difference in the two comes from physical imperfections in the measuring equipment and the ability to follow the measurement procedure.

In the surface finish example, the probe tip may not be perfectly spherical and the guideways built in the instrument may not be perfectly straight. The error that we get from these deviations is the implementation uncertainty. Calibrations and prescribed measurement procedures are used to limit the implementation uncertainty.

The operator model is used to search for uncertainty contributors. First, it is necessary to look for ambiguities in the conceptual, perfect operator. Then, differences between the conceptual, perfect operator and the conceptual, intended operator must be identified. Finally, a manufacturer identifies implementation differences between the conceptual, intended operator and the actual operator. This provides a list of uncertainty
contributors that can be quantified and used for the development of an uncertainty budget.

## Using the techniques

Uncertainty budgets help a manufacturer understand the quality of measurements. Depending on the particular situation, there may need to be a very refined budget, or the manufacturer may just want to know, on an order of magnitude, how good the measuring process is.

Using the techniques presented here, it is possible to successfully identify the three to five largest uncertainty contributors for a measurement process.

## Quantifying uncertainty contributors

Once uncertainty contributors have been identified, the next step is putting numbers to the contributors so they can fit into an uncertainty budget.

The "Guide to the Expression of Uncertainty in Measurement" offers two different ways of evaluating uncertainty contributors: type A and type $B$ evaluations. Type A evaluations use statistical
 tools to find the experimental standard deviation
from a series of observations. Type B evaluations use other means to determine an equivalent standard deviation.

Type A evaluations include an experiment that allows observation of the variations caused by the uncertainty contributor. Then the variation is analyzed by statistical means to find the experimental standard deviation for the contributor. The limitation in type A evaluations is that all the variation the contributor causes must be observed. This means the variation must be sampled often enough to capture the fastest variation and long enough to capture the slowest variation. Type A evaluations can't be used for contributors that cause a constant offset error.

The type A evaluation is the standard technique traditionally used for assessing measurement uncertainty. While it is not really an uncertainty evaluation, a gage repeatability and reproducibility study is a type A study, wherein the variation is observed and treated using statistical tools. The problems with type A evaluations are that they are work intensive, there is no guarantee that all the variations a contributor causes have been seen, and there is no way to be sure that the samplings are representative for the variations the contributor may cause over time.

## Another evaluation type

The type B evaluation provides freedom to use all the information available, such as prior knowledge, manufacturer's specifications, and information from calibration certificates, to estimate uncertainty quickly and cost effectively.

Normal distribution is one way to evaluate uncertainty contributors so that they can be quantified and budgeted for. It allows a manufacturer to take into account prior knowledge, manufacturer's specifications, etc. Normal distribution helps understand the magnitude of different uncertainty factors and understand what is important.


Triangular distribution is most often used in evaluations of noise and vibration. The manufacturer must be more comfortable estimating the width of variation using "hard" limits rather than a certain number of standard deviations.


Rectangular distribution is fairly conservative. The manufacturer has an idea of the variation limits, but little idea as to the distribution of uncertainty contributors between these limits. It is often used when information is derived from calibration certificates and manufacturer's specifications.

$U$-shaped distribution is not as rare as it seems. Cyclic events, such as temperature, often yield uncertainty contributors that fall into this sine wave pattern.

Type B evaluations estimate the limits of the variations caused by an uncertainty contributor, assumes a distribution for the variation between these limits, and uses this information to calculate an equivalent standard deviation. The four most commonly used distributions are: normal, triangular, rectangular, and U-shaped.

The normal distribution is used when there is a better probability of finding values closer to the mean value than further away from it, and one is comfortable in estimating the width of the variation by estimating a certain number of standard deviations.

An example of normal distribution is the speed of cars on a highway. If there is a speed limit of 65 mph , and the few percent that are going very fast or very slow are ignored, it can be decided that $95 \%$ of the cars go between 55 and 75 mph . Because $\pm 2$ standard deviations of a normal distribution covers $95 \%$ of the
distribution, it's found that 75 to $55 \mathrm{mph}=20 \mathrm{mph}$ is 4 standard deviations- $\pm 2$ standard deviations. Therefore, the equivalent standard deviation is $20 \mathrm{mph} 44=$ 5 mph .

It is easy to see how quickly this estimate can be made, compared to the effort involved in actually measuring the car speed with a radar gun over a period of time long enough to be representative of the overall variation. While the estimate may not be accurate to more than 20 to $30 \%$, it is good enough to help understand the relative magnitude of different uncertainty contributors and help understand what is truly important in keeping the measurement process under control.

Triangular distribution is used when it is known that there is a better probability of finding values close to the mean value than further away from it, and one is more comfortable estimating the width of the variation by estimating "hard" limits rather than a certain number of standard deviations.

Typical examples of where triangular distribution is used are noise and vibration. The relationship between the equivalent standard deviation, s , and the variation limits, $\pm \mathrm{a}$, is:
$\mathrm{s}=\mathrm{a} / \mathrm{sqrt}(6)$ or approximately 0.4 a
Rectangular distribution is used when the variation limits are known, but there is no information about the distribution between these limits. This is typically the case with the information found in calibration certificates, especially certificates of compliance, and manufacturers' specifications.

The relationship between the equivalent standard deviation, s , and the variation limits, $\pm \mathrm{a}$, for rectangular distribution is:
$\mathrm{s}=\mathrm{a} / \operatorname{sqrt}(3)$ or approximately 0.6 a
The conversion factor for rectangular distribution is larger than the one for triangular distribution. This means that if there is doubt whether rectangular distribution or triangular distribution is the best assumption for a particular contributor, rectangular distribution is the more conservative assumption. This leads to a higher equivalent standard deviation value for the same variation width, $\pm$ a.

U-shaped distribution is used when it is known there is a better probability of finding values close to the variation limits than around the mean value. While this type of distribution may seem rare at first glance, it is fairly common, because $U$ shaped distribution is the probability density function for a sine wave. Cyclic influences, such as temperature variation, usually follow a sine wave pattern. Temperature influences are often the limiting factor in dimensional metrology, so the U-shaped distribution is important to know.

The relationship between the equivalent standard deviation, s , and the variation limits, $\pm \mathrm{a}$, for U -shaped distribution is:
$\mathrm{s}=\mathrm{a} / \operatorname{sqrt}(2)$ or approximately 0.7 a

The conversion factor is even larger for U-shaped distribution than it is for rectangular distribution. This means the U -shaped distribution is an even more conservative assumption than rectangular distribution, because it returns a higher equivalent standard deviation value for the same variation width, $\pm$ a.

## Choosing the right technique

The choice between type A and type B evaluation depends on what information is available. The type A evaluation is attractive if experimental data is available. Otherwise, type B is faster and easier to use, especially in a first draft uncertainty budget where a manufacturer tries to get a feel for the relative size of the contributors. It's not a good use of time and effort to conduct an elaborate type A evaluation of one contributor, only to find that another contributor is an order of magnitude larger. So, start with type B evaluations and after the relative size of the contributors is understood, change some of the major contributors to type A if there is not enough confidence in the initial type B estimate.

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## The Circle of Contributors

The environment. The uncertainty contributors related to the environment are primarily temperature and vibration. In some cases humidity and noise, both acoustic and electrical, also will contribute to the uncertainty.

Reference element of the measuring equipment. Broken down into its basic elements, most measurements consist of a measuring object, a reference element, and some equipment to compare the two. In a micrometer, the spindle is the reference element. In a gage block comparator, the known gage block is the reference element. Looking at the reference element, separately, often gives a clearer picture of where the uncertainty originates, than looking at the measuring equipment as one item.

The measuring equipment. Using this division, the measuring equipment is the apparatus that compares the measuring object to the reference element. It is the LVDT probe and the base in the gage block comparator. It is the body and the anvils on the micrometer.

Measurement setup. This is the fixturing and tools that are used to hold the measuring object in the measuring equipment, as well as the "foundation" for the measuring equipment. It is the plate holding the gage blocks and the vibration isolated base in the gage block comparator. It is the benchtop fixture that may be used to hold the micrometer.

Software and calculations. Software contributes to the uncertainty, if the equations used are different from what is intended in the measurement. For example, least square algorithms may be used instead of minimum
zone algorithms because they are faster and converge better, but they do give a different answer than what is called for. Software algorithms may not be solid and break down when presented with unexpected data. Finally, there are plain hand-made calculations that can go wrong and cause uncertainty.

Metrologist. The human influence is not limited to the body heat and dexterity of the operator. There are also influences from how the measuring process is designed ergonomically. Is there enough light? Is it easy to take the necessary readings? Does the environment encourage accurate work?

Measuring object. The influence from the measuring object is often significant, but may be overlooked as a contributor to the uncertainty. For example, when measuring the diameter of a pin, the roundness, straightness, taper, and roughness of the pin are generally the limitations of determining the diameter of the pin.

Definition of the measurand. The measurand is what is to be measured. It may be the diameter of a pin or the length of a gage block. Given just a cursory glance, these definitions seem self evident, but when trying to measure to a level of accuracy, where the form error comes into play, the slight or not so slight ambiguities in the definition of the measurand may become significant. It's important to think about what is really being measured.

Measuring procedure. The measuring procedure determines how long the work piece is allowed to acclimate before it is measured, or how a sequence of repeat measurements are arranged to compensate for drift. Both go to the uncertainty budget.

Physical constants. Typically, physical constants, such as the thermal expansion coefficient of steel, are used from reference books. However, when using the book value to make the compensation, there will be an uncertainty, because the particular batch of steel used to make each gage block has a slightly different expansion coefficient.

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